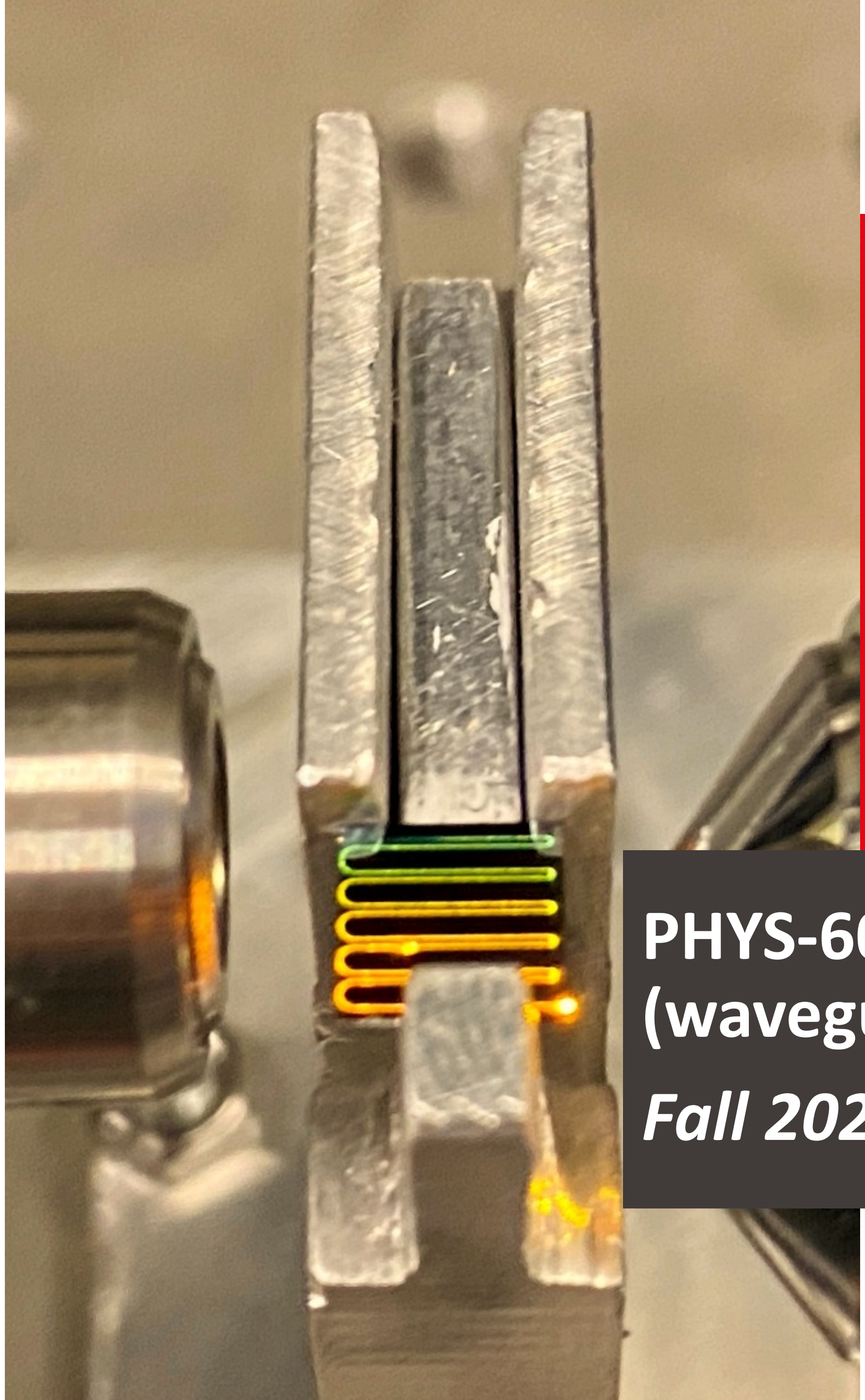
EPFL



Optical Kerr effect

Module 8 – Parametric amplification

PHYS-607 : Nonlinear fibre (waveguide) optics

Fall 2022

Prof. Camille Brès - camille.bres@epfl.ch

Ecole
Polytechnique
Fédérale de
Lausanne

Parametric amplifiers

The parametric gain can be used to make optical amplifier

- These FWM based devices are called optical parametric amplifier (OPA)

If the amplifier is places within an optical cavity providing feedback periodically, we now have a parametric oscillator (OPO)

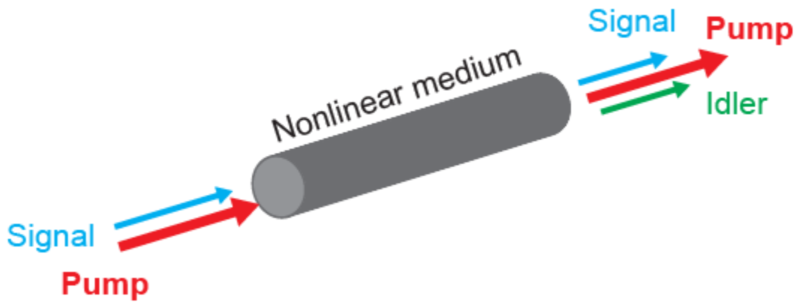
The interest in OPAs comes from

- Large potential bandwidth (> 100 nm)
- High gain: > 40 dB
- Fast nonlinear response enable a variety of signal processing applications

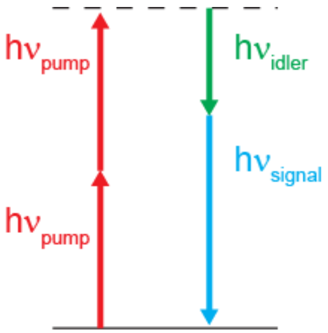
An important property of an amplifier is its bandwidth over which the gain is relatively uniform

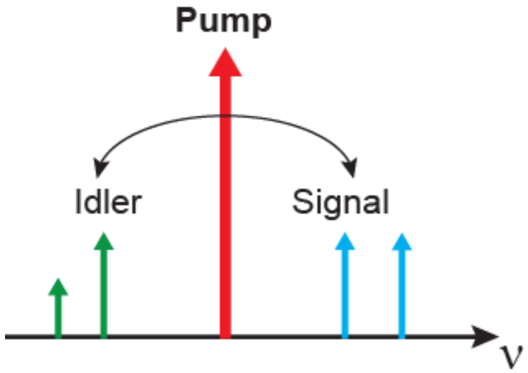
FWM summary so far

Reminder



$$2\nu_{pump} \longrightarrow \nu_{signal} + \nu_{idler}$$





Phase mismatch:

$$\kappa = \Delta k + 2\gamma P_0 \approx \beta_2 (\Omega_S)^2 + \frac{\beta_4}{12} (\Omega_S)^4 + 2\gamma P_0$$

Parametric gain:

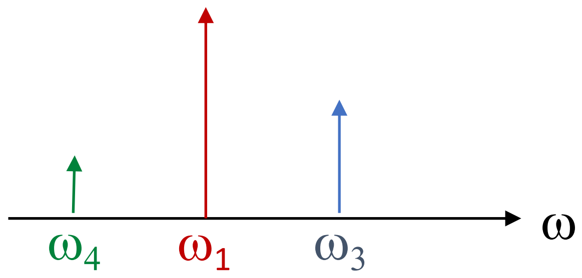
$$g = \sqrt{(\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2}$$

β_n : n^{th} order dispersion
γ: nonlinear coefficient
Ω_s: frequency offset from pump

Parametric processes are *phase sensitive*

- Interaction depends on the relative phases of all light beams
- Effects can only accumulate if *phase matching* condition is satisfied
  - If frequencies involved are close to each other
  - If the chromatic dispersion profile has a suitable shape

FWM summary so far



Reminder

Approximate analytic solution under undepleted pump assumption

$$B_3(z) = (a_3 e^{gz} + b_3 e^{-gz}) \exp\left(-j\kappa \frac{z}{2}\right)$$

$$B_4^*(z) = (a_4 e^{gz} + b_4 e^{-gz}) \exp\left(j\kappa \frac{z}{2}\right)$$

$$\frac{dB_3}{dz} = 2j\gamma\sqrt{P_1 P_2} \exp(-j\kappa z) B_4^*$$

$$\frac{dB_4^*}{dz} = -2j\gamma\sqrt{P_1 P_2} \exp(j\kappa z) B_3$$

a_3, b_3, a_4 and b_4 are determined from boundary conditions

Gain spectrum and bandwidth

We assume that the signal wave at ω_3 is launched at the input of the amplifier together with a single pump at ω_1

- Practical situation for most OPA
- Can also launch the 'idler' wave at ω_4 at the input of the OPA (can then obtain phase sensitive amplification) – more complex analysis

$$B_3(0) = a_3 + b_3$$

$$B_4^*(0) = 0$$

$$\frac{dB_3}{dz} = 2j\gamma\sqrt{P_1P_2}\exp(-j\kappa z)B_4^*$$

- Solving the equations, we obtain (similar for a_4 and b_4)

$$a_3 = \frac{1}{2}\left(1 + j\frac{\kappa}{2g}\right)B_3(0) \qquad b_3 = \frac{1}{2}\left(1 - j\frac{\kappa}{2g}\right)B_3(0)$$

Power output

Putting back a_3, b_3, a_4 and b_4 in the approximate analytic solutions:

$$B_3(z) = (a_3 e^{gz} + b_3 e^{-gz}) \exp\left(-j\kappa \frac{z}{2}\right)$$

$$\Downarrow$$

$$B_3(z) = B_3(0)\left[\cosh(gz) + j\frac{\kappa}{2g}\sinh(gz)\right]\exp\left(-j\kappa\frac{z}{2}\right)$$

The signal power $P_3 = |B_3|^2$ grows with z as :

$$P_3(z) = P_3(0)\left[1 + \left(1 + \frac{\kappa^2}{4g^2}\right)\sinh^2(gz)\right]$$

The Idler power is such that $P_4(z) = P_3(z) - P_3(0)$:

$$P_4(z) = P_3(0)\left[\left(1 + \frac{\kappa^2}{4g^2}\right)\sinh^2(gz)\right]$$

$$g = \sqrt{(\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2}$$

$$P_0 = \frac{P_1 + P_2}{2}$$

$$r = \frac{\sqrt{P_1 P_2}}{P_0}$$

$$\kappa = \Delta k + 2\gamma P_0$$

Amplification factor

The amplification factor is :

$$G_s = \frac{P_3(L)}{P_3(0)} = 1 + \left(1 + \frac{\kappa^2}{4g^2}\right)\sinh^2(gL)$$

$$G_s = \frac{P_3(L)}{P_3(0)} = 1 + \left(\frac{\gamma P_0 r}{g}\right)^2 \sinh^2(gL)$$

It depends on the phase mismatch κ.

- Can become quite small if phase matching is not achieved

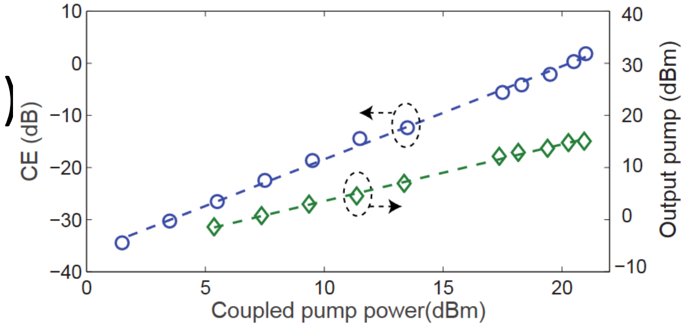
Note that the conversion efficiency CE is : $CE = \frac{P_4(L)}{P_3(0)} = \left(\frac{\gamma P_0 r}{g}\right)^2 \sinh^2(gL)$

Non-phase-matched behavior

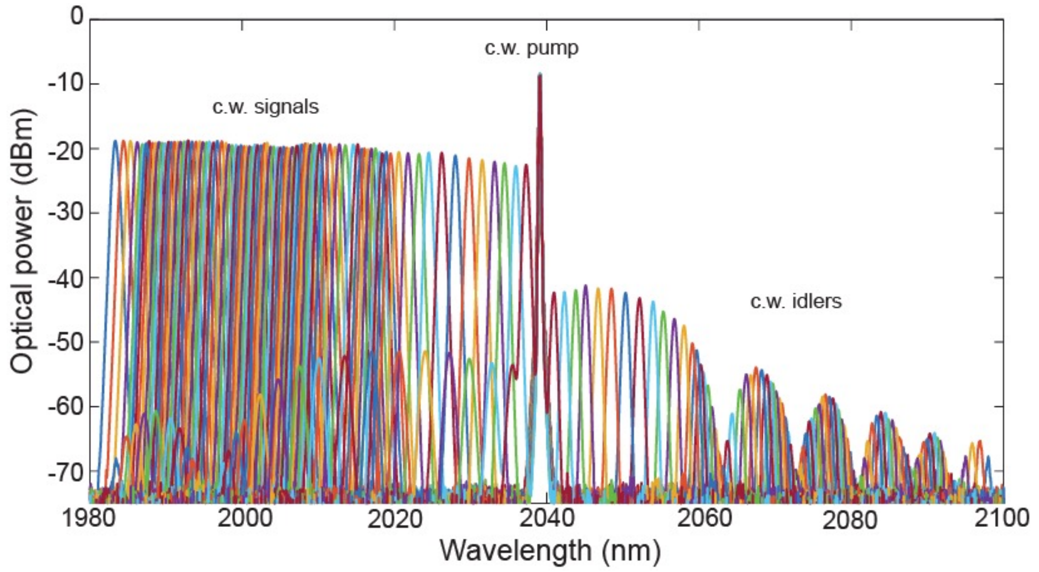
Consider the limit where linear phase mismatch Δk, strongly dominates nonlinear phase shift:

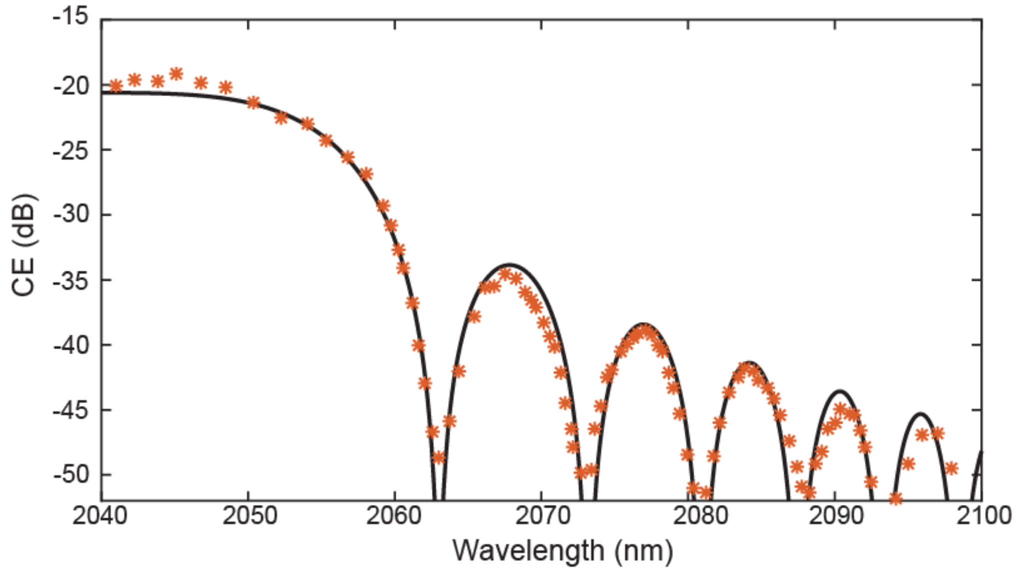
$$\Delta k \gg 2\gamma P_0 \Longrightarrow g = \sqrt{(\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2} \;\; is\ imaginary$$

The amplification factor reduces to a *sinc* function (single pump case here)



$$G_s \approx 1 + (\gamma P_0 L)^2 \,\mathrm{sinc}^2\left(\frac{\kappa L}{2}\right)$$





Phase-matched behavior

Let's consider the case of perfect phase matching: $\kappa = 0$ and $gL >> 1$

The amplifier gain can be approximated as:

$$G_s \approx \frac{1}{4}\exp(\gamma P_0 L)$$

Reminder

Recall that for *the single pump case* $\kappa = \Delta k + 2\gamma P_0 \approx \beta_2(\Omega_S)^2 + \frac{\beta_4}{12}(\Omega_S)^4 + 2\gamma P_0$

Maximum gain is when $\kappa = 0$, i.e. for $\Delta k = -2\gamma P_0$

Gain exists when $g = \sqrt{(\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2}$ is positive, meaning when $-4\gamma P_0 < \Delta k < 0$

Single pump peak of the parametric gain

We need $\Delta k = -2\gamma P_0$, meaning:

$$\beta_2\left(\Omega_{S,peak}\right)^2 + \frac{\beta_4}{12}\left(\Omega_{S,peak}\right)^4 = -2\gamma P_0$$

If second order dispersion dominates: $\beta_2\left(\Omega_{S,peak}\right)^2 = -2\gamma P_0$

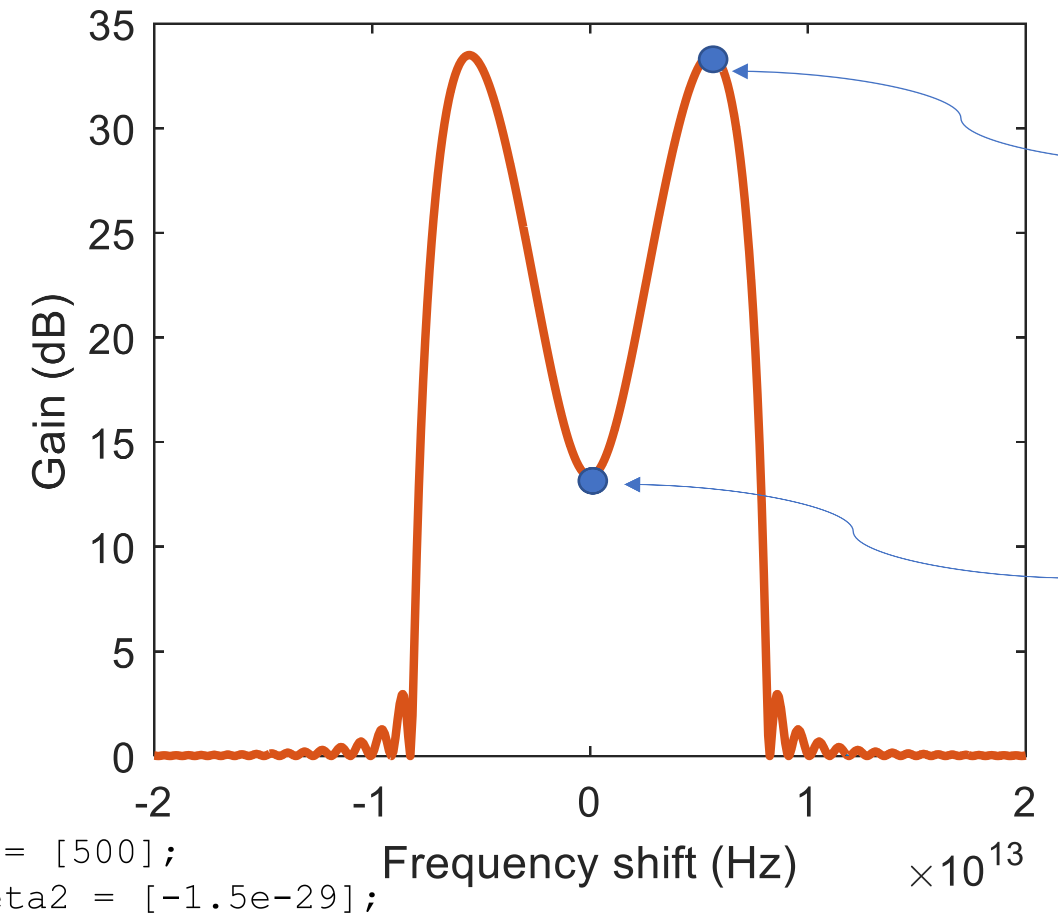
- Note that in order to satisfy such condition, the pump must be in the anomalous dispersion ($\beta_2 < 0$)

Peak therefore occurs at $\Omega_{S,peak} = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$

Similar to MI: MI ⇔ phase matched FWM !

Typical gain curve in anomalous dispersion – single pump case

$$G_s = \frac{P_3(L)}{P_3(0)} = 1 + \left(\frac{\gamma P_0}{g}\right)^2 \sinh^2(gL)$$



➔ Exponential amplification @ $\Omega_{s,peak} = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$

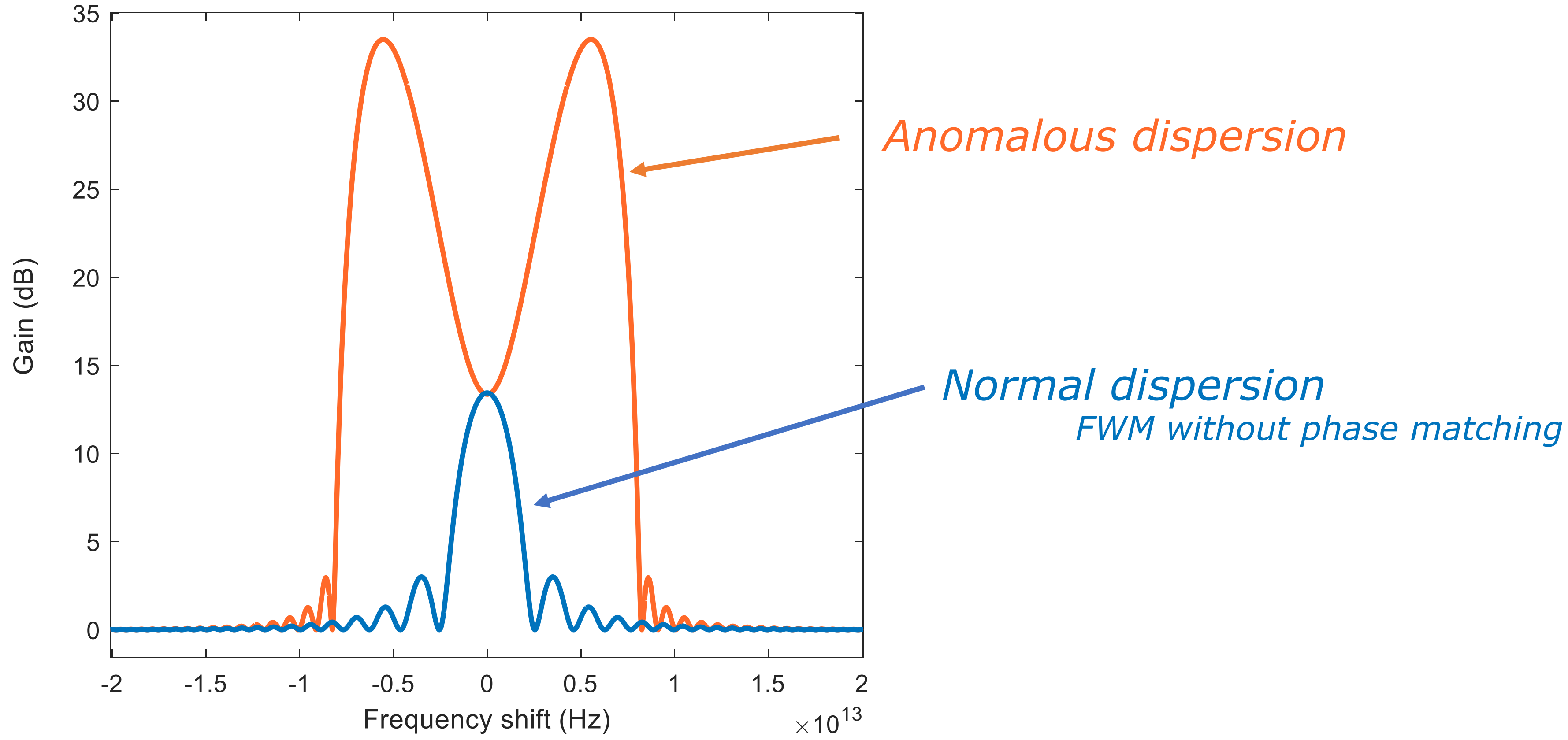
$$G_s \cong \frac{1}{4}\exp(2\gamma P_0 L)$$

➔ Quadratic amplification @ Ω->0

$$G_s \cong 1 + (\gamma P_0 L)^2$$

```
L = [500];
Beta2 = [-1.5e-29];
gamma= [0.013];
P = [0.7];
```

Typical gain curves



We can still have parametric gain when the pump is located in the normal dispersion ($\beta_2 > 0$)

- Requires a negative β_4
- Since β_4 is typically much smaller, the gain will happen at a much larger detuning and over a much narrower bandwidth.

Gain bandwidth

In the case of long fibres, the gain bandwidth $\Delta\Omega$ is set by the fiber length

- It can be estimated as the bandwidth for which $|\kappa L| = \pi$. This is because the gain is reduced roughly by a factor of 2.

In the small-gain limit (i.e. $2\gamma P_0 L << \pi$):

$$\Delta\Omega \approx \left[\frac{4\pi}{|\beta_2|L}\right]^{\frac{1}{2}}$$

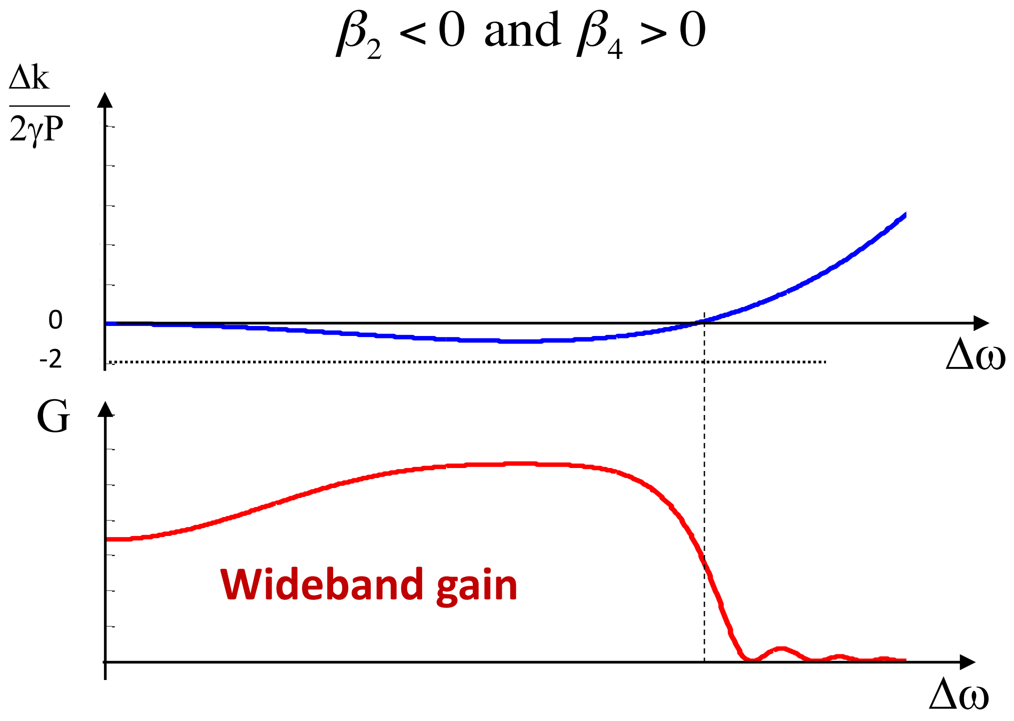
For large gain with $\Delta\Omega < \Omega_{s,\ peak}$

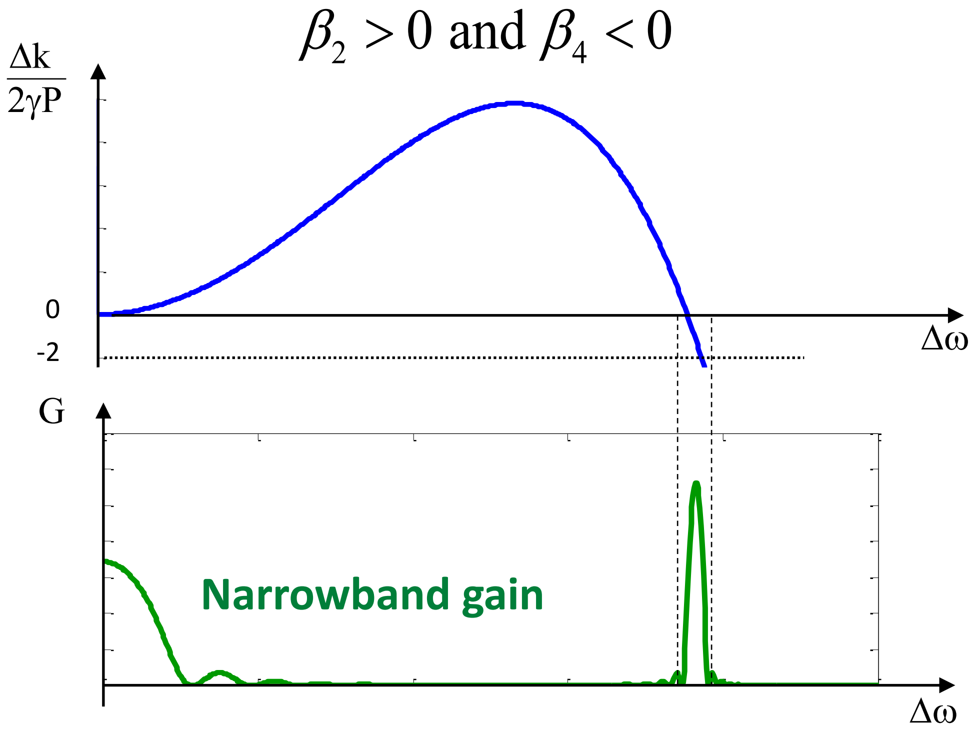
$$\Delta\Omega \approx \frac{\pi}{L}\left[\frac{1}{2\gamma|\beta_2|P_0}\right]^{\frac{1}{2}}$$

Consideration on the conversion ‘reach’

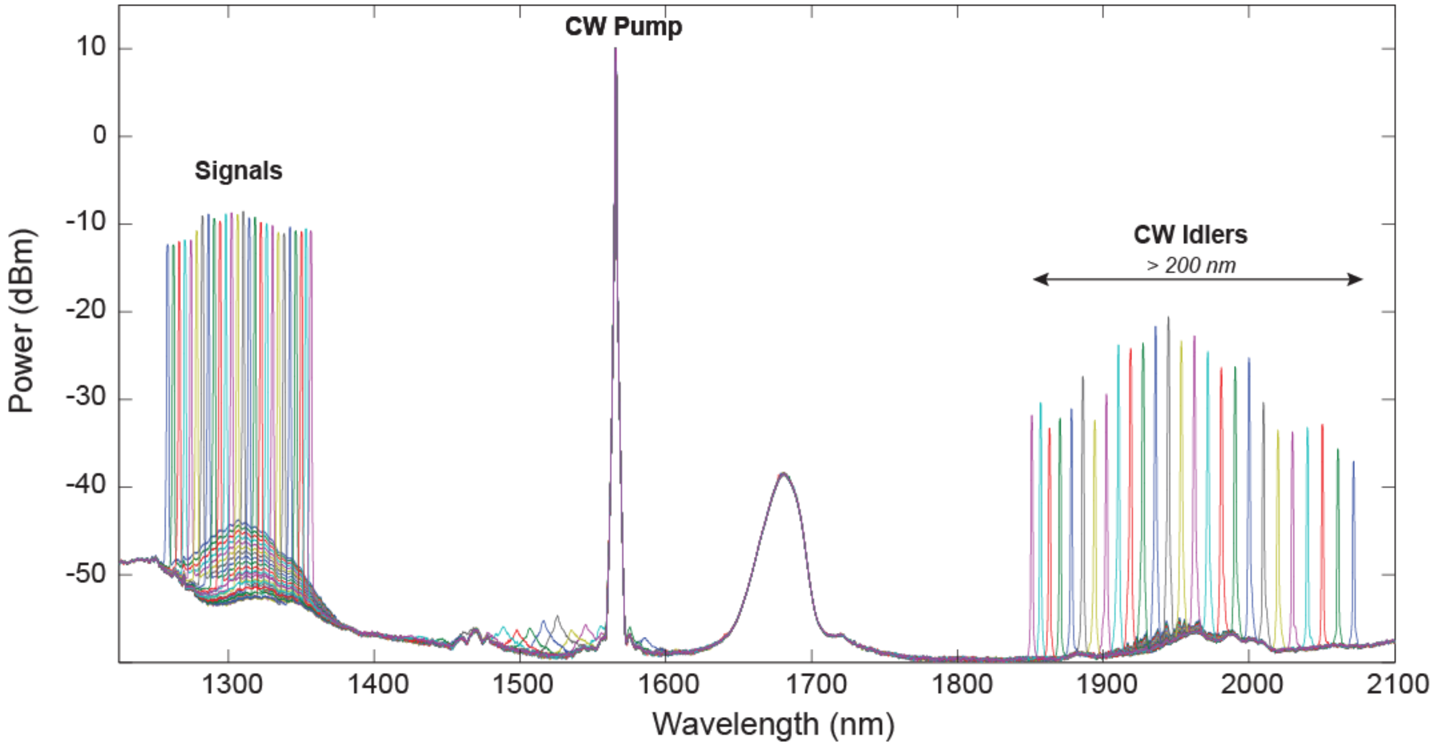
Parametric conversion condition: $-4\gamma P_0 < \beta_2(\Omega_S)^2 + \frac{\beta_4}{12}(\Omega_S)^4 < 0$ ***+ Low loss, nonlinear medium***

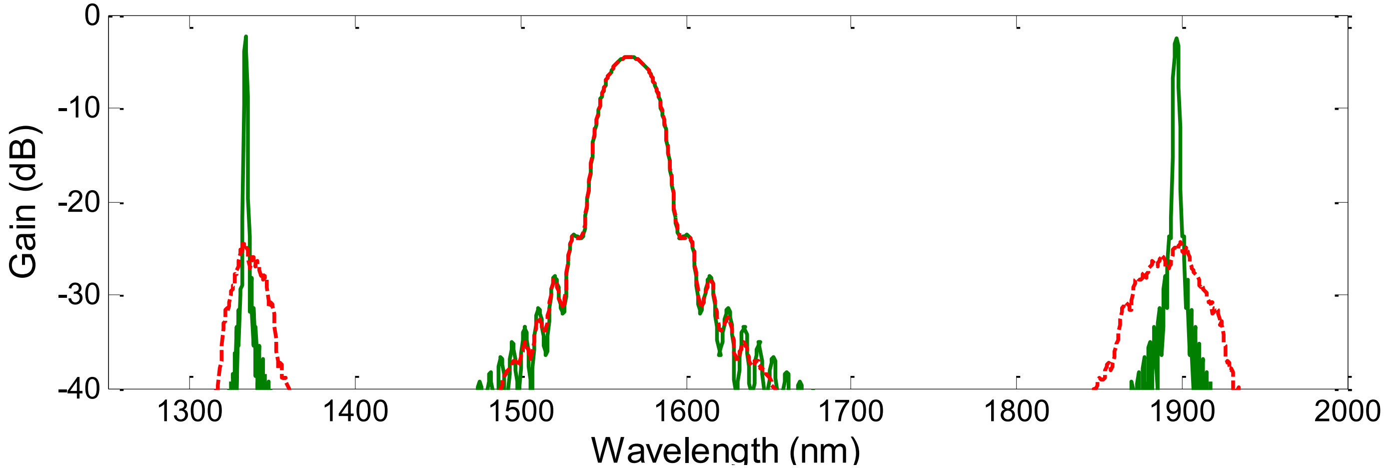
Amplification factor: $G_s = \frac{P_3(L)}{P_3(0)} = 1 + \left(\frac{\gamma P_0}{g}\right)^2 \sinh^2(gL)$



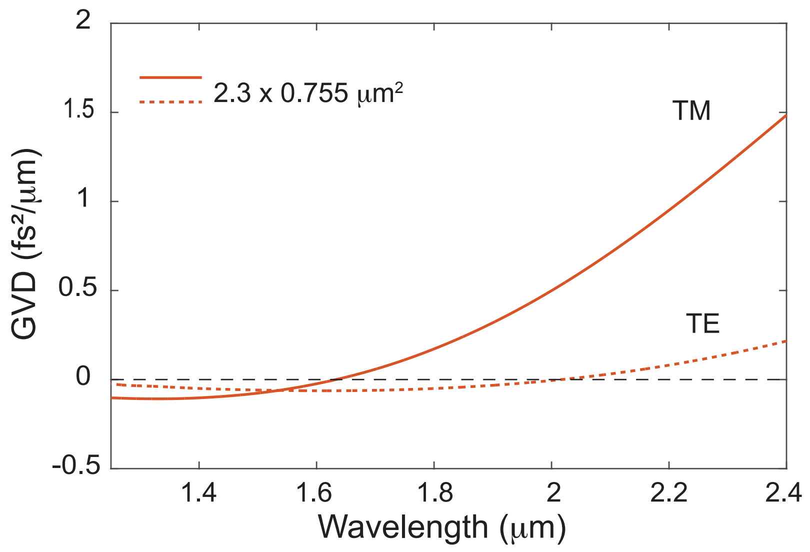


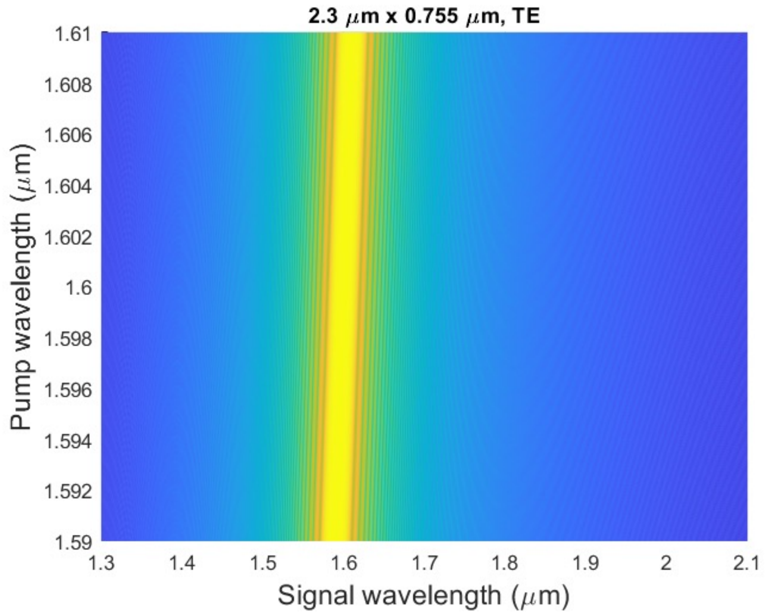
Normal dispersion parametric conversion – effect of dispersion fluctuations

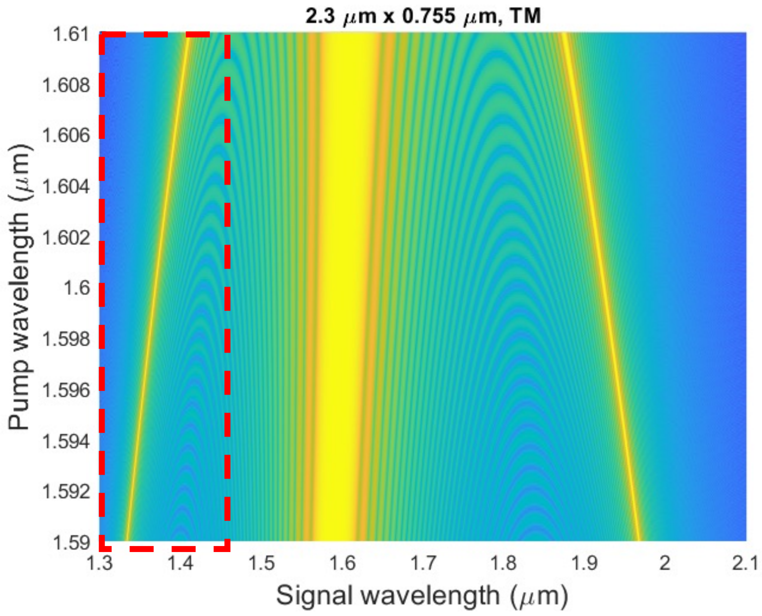


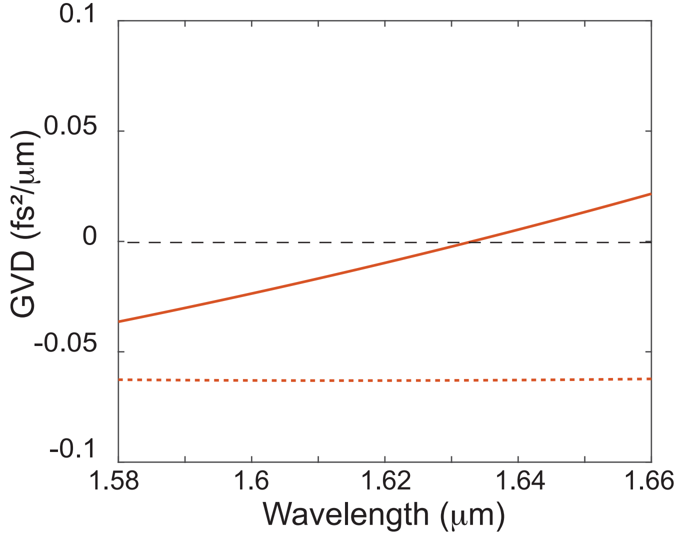


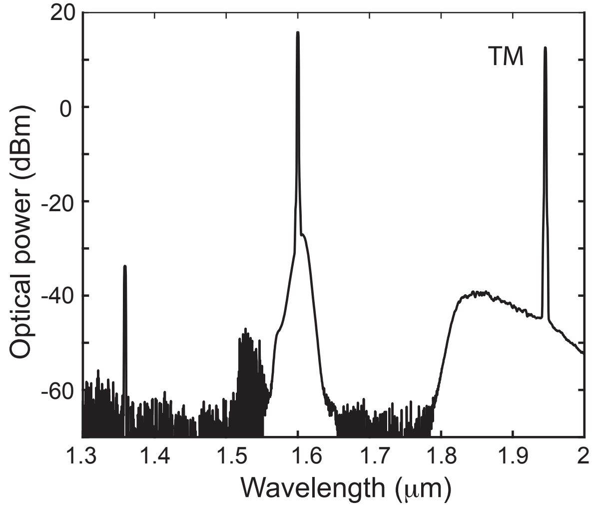
Another illustration

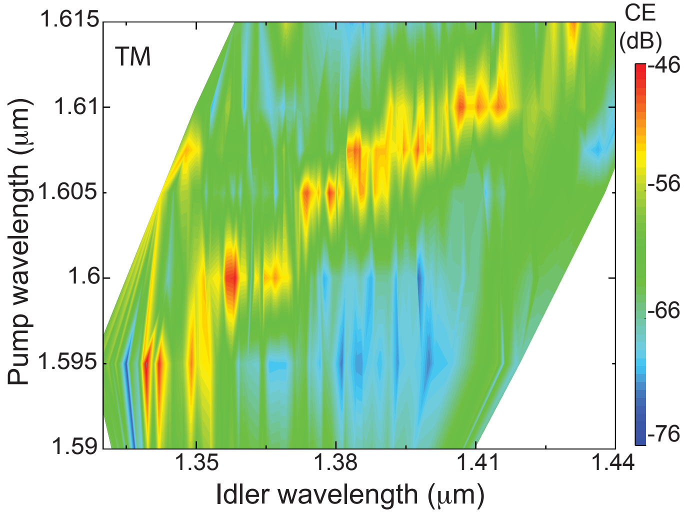










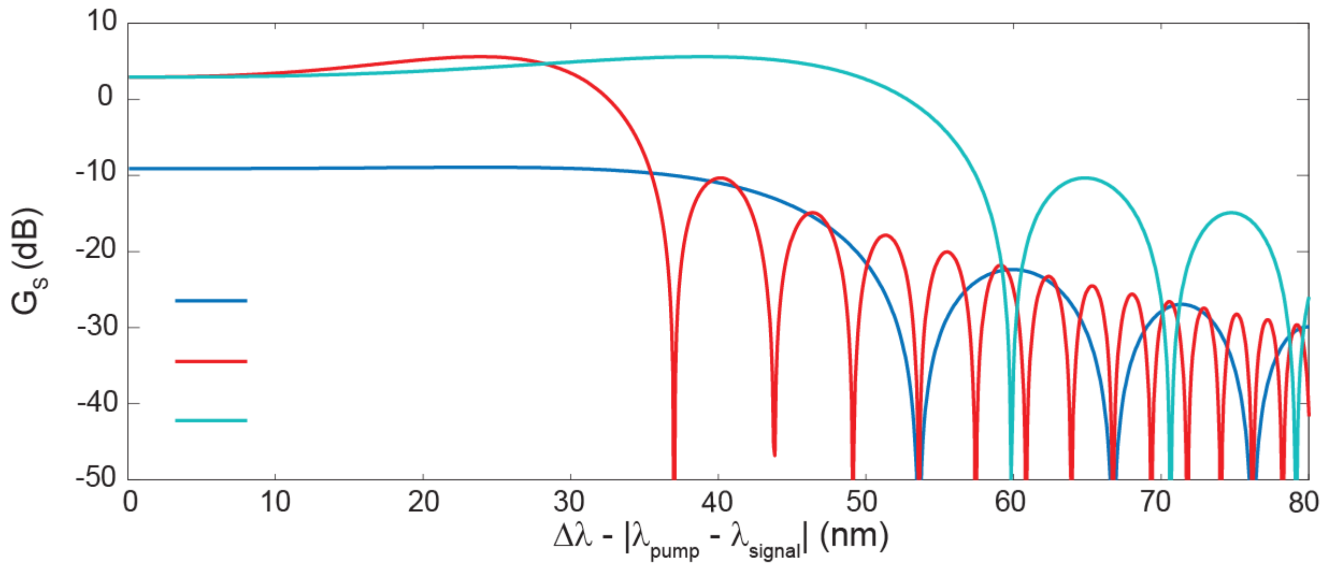


Quiz

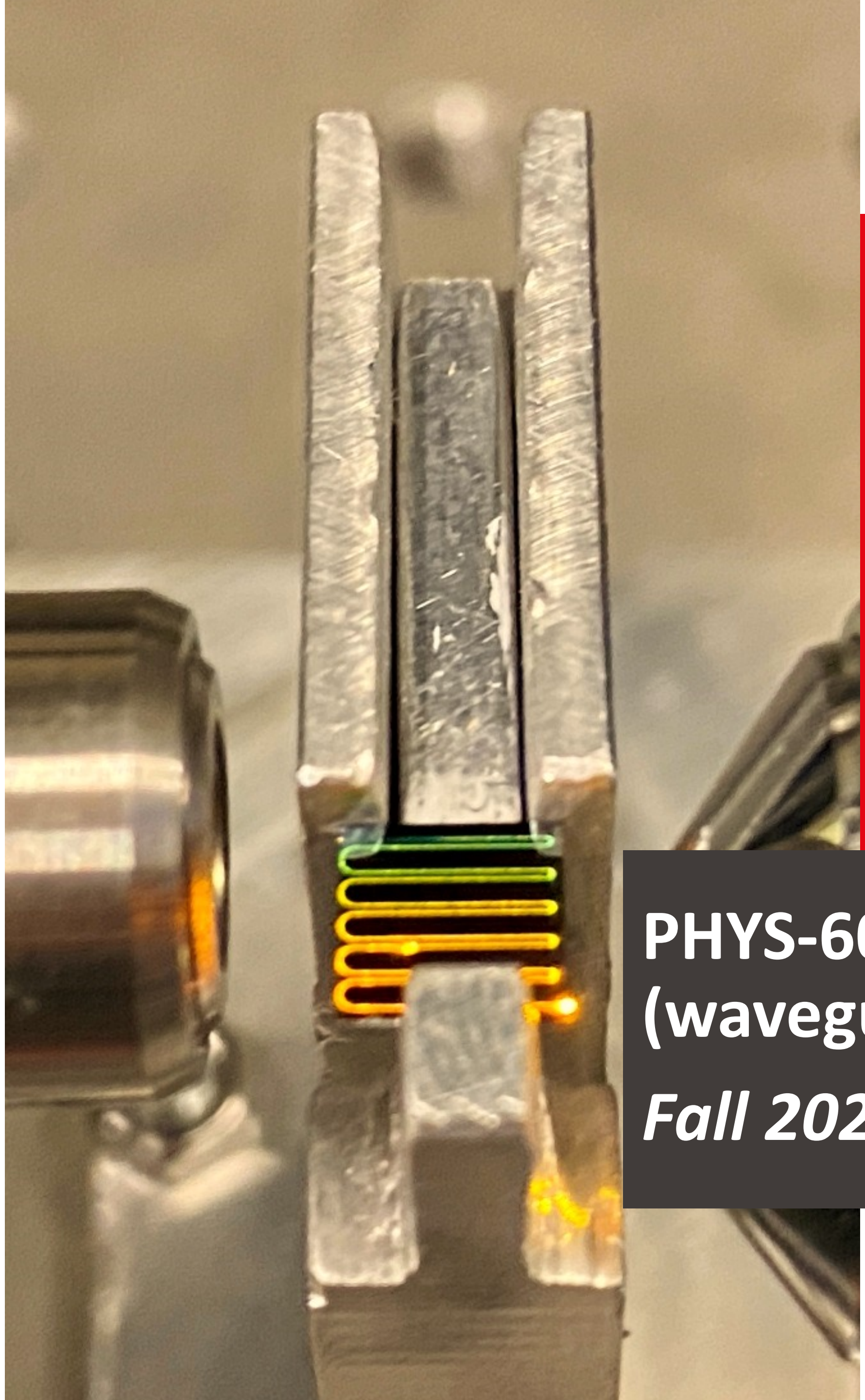
Two different lengths (5 mm and 20 mm) of the same waveguide are used for parametric amplification using the same pump (degenerated case). Three different combinations of length/pump power are tested:

- L = 5 mm and P = 350 mW
- L = 20 mm and P = 350 mW
- L = 5 mm and P = 1400 mW

Match these combinations to parametric gain measured as a function of signal detuning:



EPFL



Optical Kerr effect

Module 9 – Supercontinuum

PHYS-607 : Nonlinear fibre (waveguide) optics

Fall 2022

Ecole Polytechnique Fédérale de Lausanne

Prof. Camille Brès - camille.bres@epfl.ch

What is a supercontinuum

Narrowband field that experiences massive continuous spectral broadening in a nonlinear medium

The supercontinuum is 50 years old

VOLUME 24, NUMBER 11 PHYSICAL REVIEW LETTERS 16 MARCH 1970

OBSERVATION OF SELF-PHASE MODULATION AND SMALL-SCALE FILAMENTS IN CRYSTALS AND GLASSES

R. R. Alfano* and S. L. Shapiro
Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360
(Received 10 December 1969)

EMISSION IN THE REGION 4000 TO 7000 Å VIA FOUR-PHOTON COUPLING IN GLASS

R. R. Alfano and S. L. Shapiro
Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360
(Received 9 January 1970)

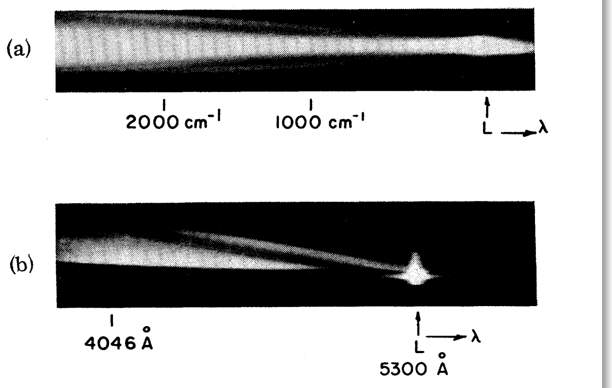


FIG. 1. (a) Anti-Stokes emission with main laser beam at slit center. Light in center is frequency swept light while outer curves is due to four-photon emission. (b) Entire angular anti-Stokes emission curve from 4000 to 5300 Å.

In bulk

Difficulties and issues with generating SC in bulk

Limitations

- Walk off
- Diffraction
- Strong dispersion resulting in limited broadening
- There is a need for very high energy which can easily induce damage

Waveguides are the way to go

WIDEBAND NEAR-I.R. CONTINUUM (0·7–2·1 μm) GENERATED IN LOW-LOSS OPTICAL FIBRES

Indexing terms: Nonlinear optics, Optical fibres

A wideband near-infrared continuum is obtained by nonlinear optical interactions in low-loss single-mode and multimode silica fibres pumped by pulses from a Q-switched Nd:y.a.g. laser. With 50 kW pump power coupled into a 315 m long 33 μm core-diameter low-loss multimode fibre, a continuum spanning the spectral range 0·7–2·1 μm with a total power of 12 kW is generated. In single-mode and low-mode-number fibres a similar continuum of lower power has been obtained. Such continuum sources are useful for measurement of fibre properties as well as for spectroscopic study of other optical materials.

ELECTRONICS LETTERS 7th December 1978 Vol. 14 No. 25

CHINLON LIN
V. T. NGUYEN

Bell Telephone Laboratories
Holmdel, New Jersey 07733, USA

W. G. FRENCH

Bell Telephone Laboratories
Murray Hill, New Jersey 07974, USA

1st November 1978

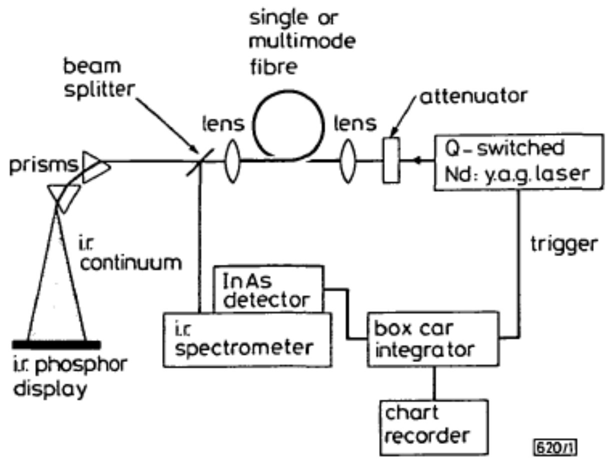


Fig. 1 *Schema of the experimental arrangement for wideband near-infrared continuum generation in optical fibres*

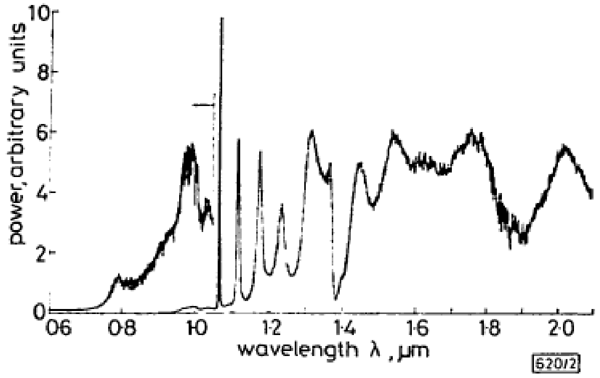


Fig. 2 *Spectra of the continuum obtained in a GeO_2 doped silica-core multimode fibre*

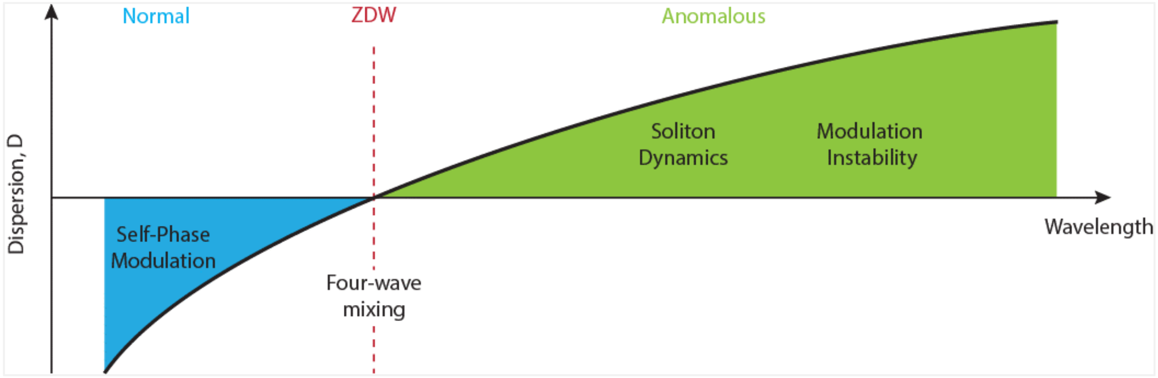
Supercontinuum in waveguides

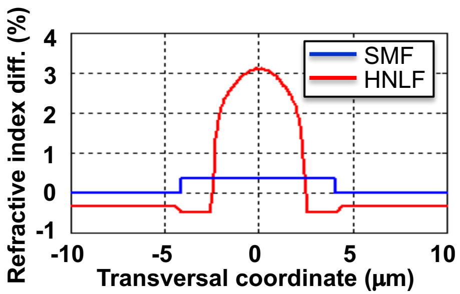
Pulse propagation in a waveguide

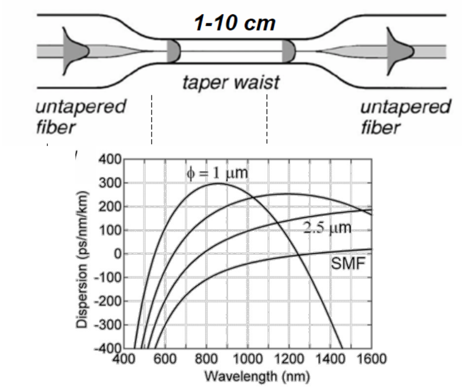
- No diffraction and long interaction length

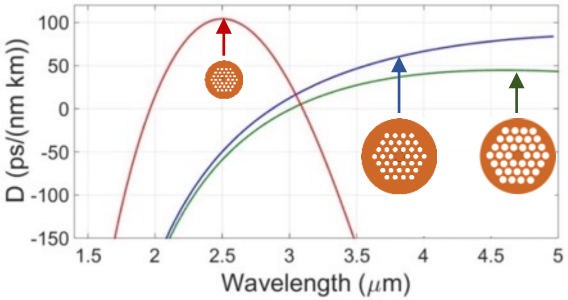
Dispersion/nonlinearity can be controlled: it is crucial

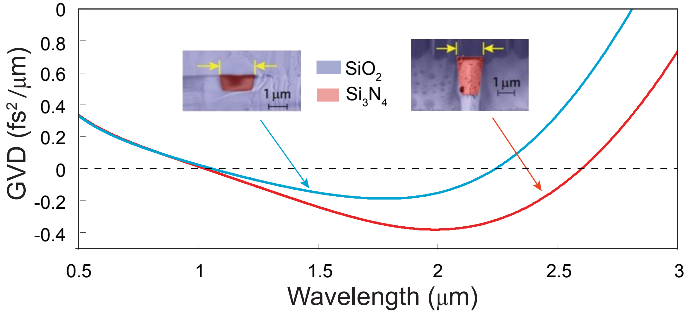
- Propagation dynamics depend on the pump wavelength relative to the zero dispersion wavelength







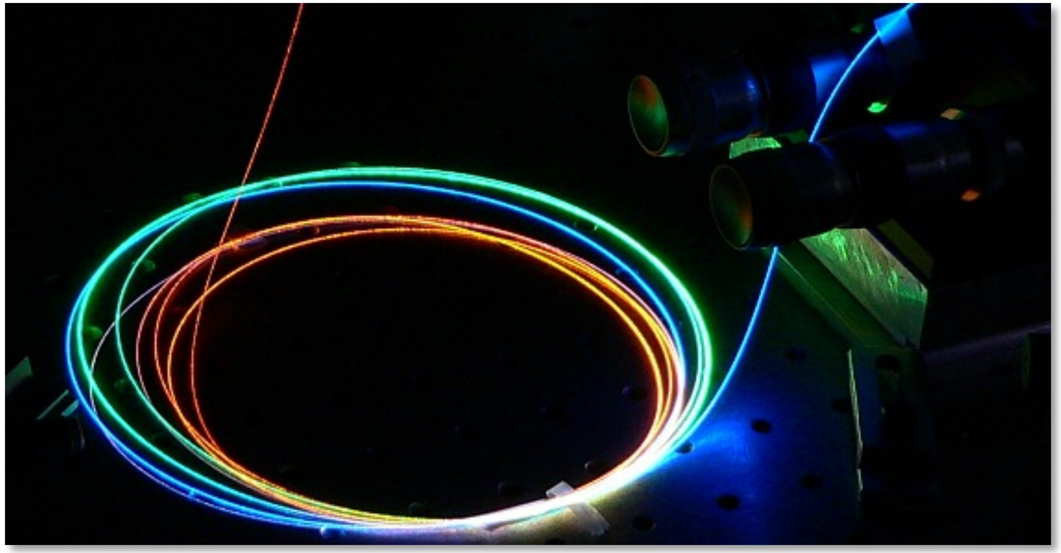


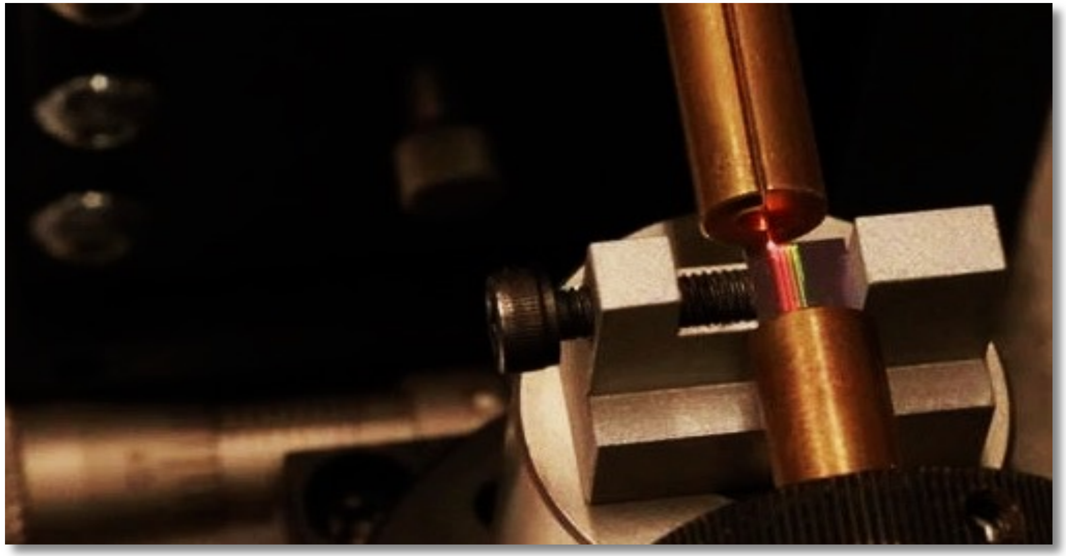


Supercontinuum 1.0

‘Anything will lase if you hit it hard enough’, **Arthur Leonard Schawlow**

Quite an easy experiment... Pretty much anything works

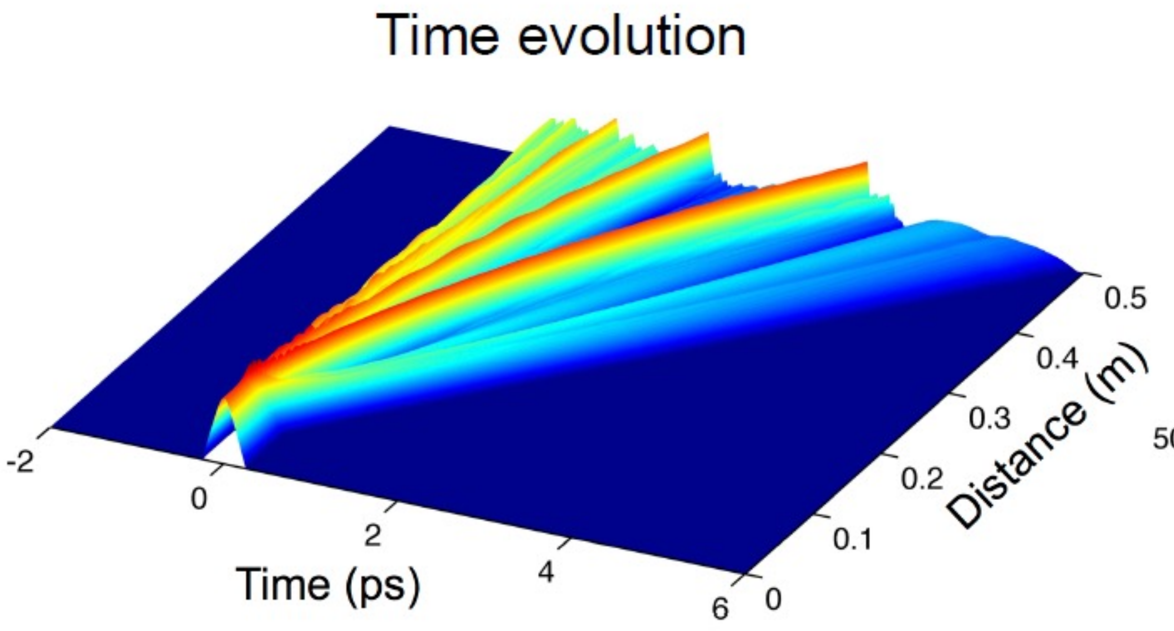


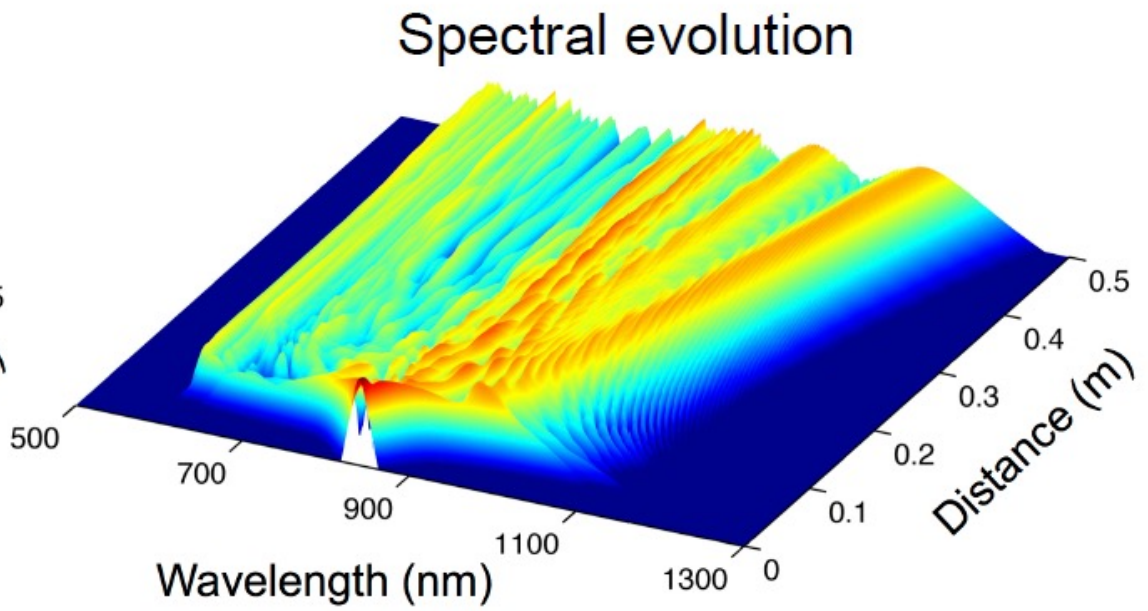


But complex system if you want to reach a specific, controlled and well defined performance.

- The physics of supercontinuum is aresult of the interplay between several nonlinear responses.

Supercontinuum generation regimes





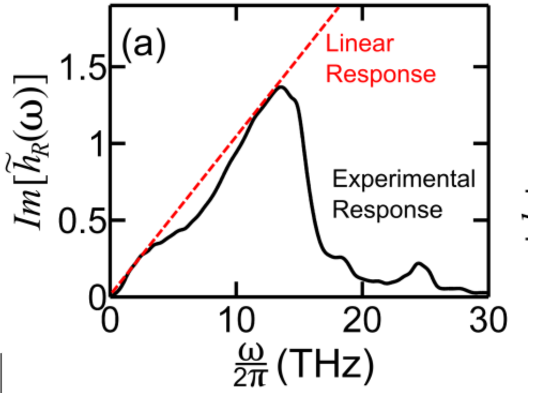
	Long pulses (ns)	Short pulses (fs)
Anomalous dispersion	Modulation instability Solitons dynamics	Solitons Dispersive waves
Normal dispersion	Raman scattering Four-wave mixing	Self-phase modulation Four-wave mixing

Modeling SCG

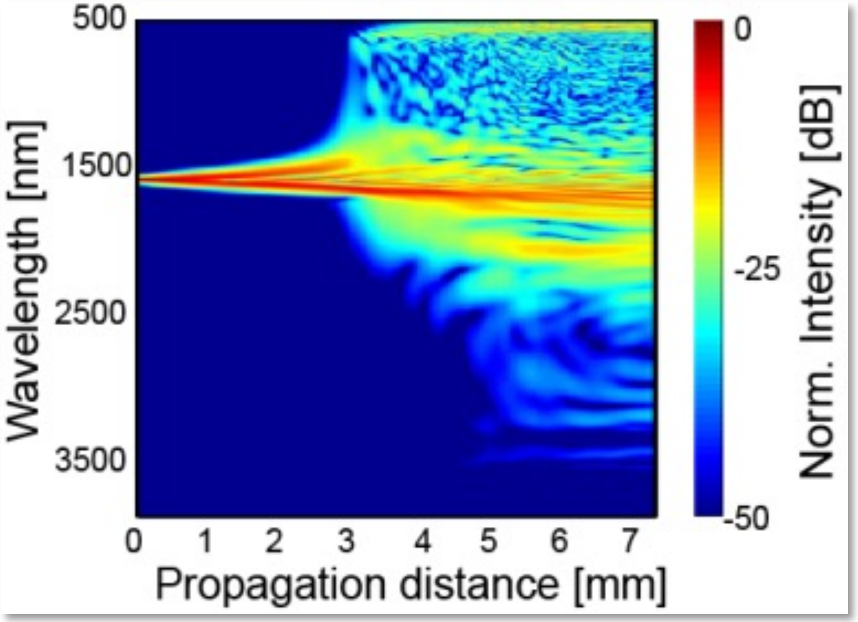
$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A + \sum_{k\geq 2} \frac{j^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} + j\left(\gamma + j\gamma_1 \frac{\partial}{\partial T}\right)\left(A(z,T)\int_0^\infty R(t')\left|A(z,T-t')\right|^2 dt'\right)$$

Important consideration

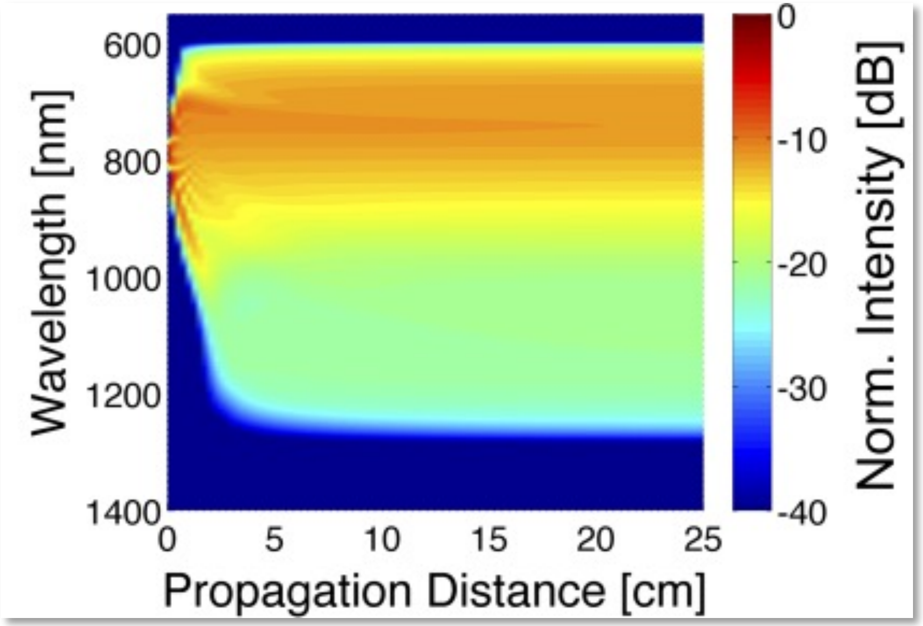
- Window size to avoid wrapping around
- Temporal and spectral resolution
- Set size (to avoid FWM artifacts)
- Use the full experimental Raman model (and not a linearized one)
- Adjust dispersion to the pumping wavelength



Quick view of supercontinuum



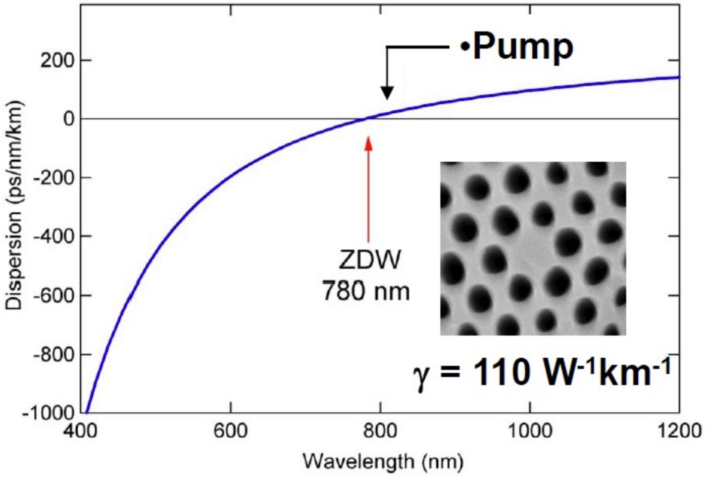
Anomalous: Soliton dynamics/MI

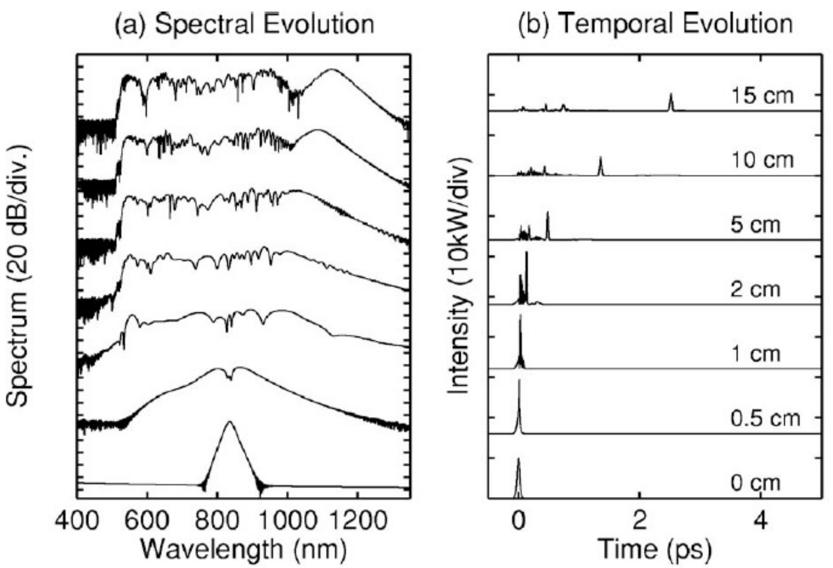


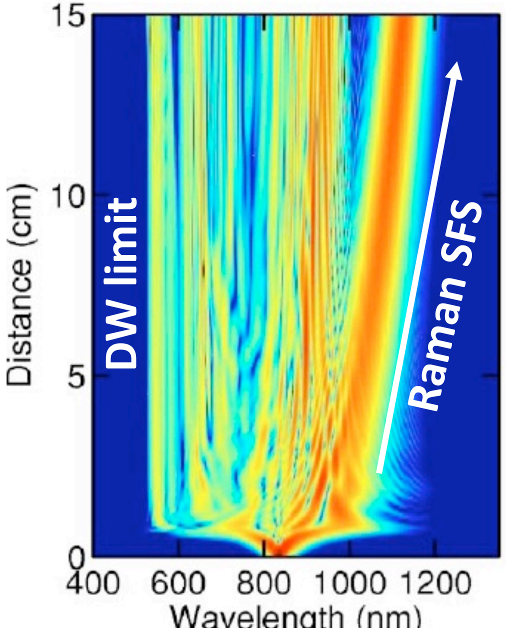
Normal: SPM/OWB

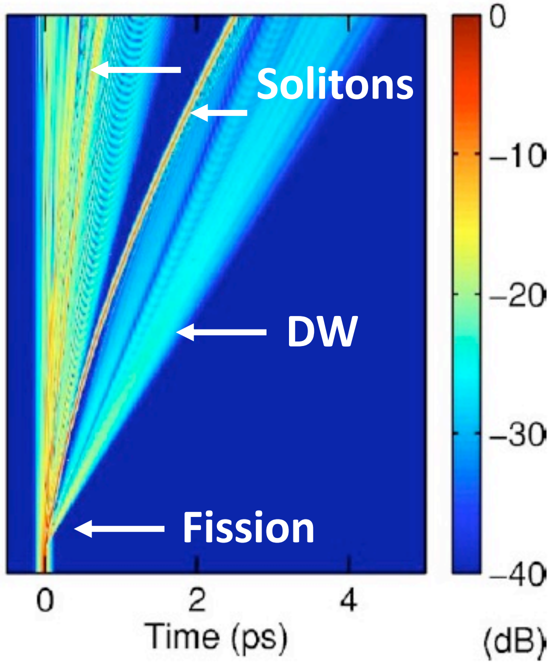
Short pulse regime and anomalous dispersion

50 fs, 10 kW input pulses







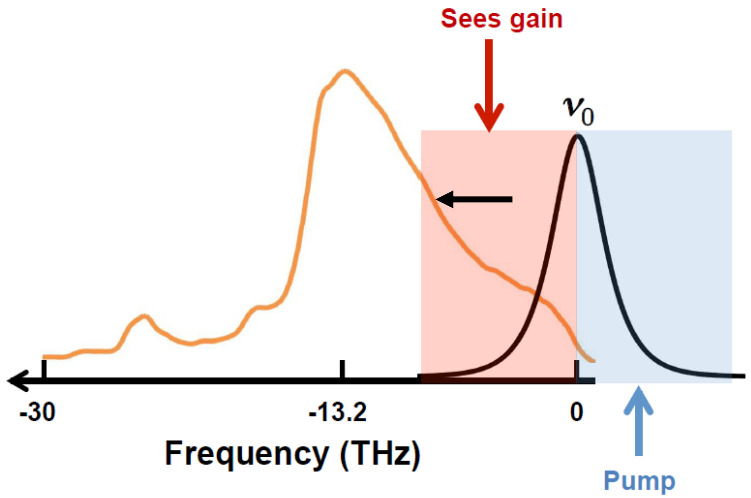


In the anomalous regime with short pulses the generation of supercontinuum comes from soliton propagation dynamics:

- Higher order soliton compression
- Soliton fission and dispersive wave generation
- Raman self frequency shift

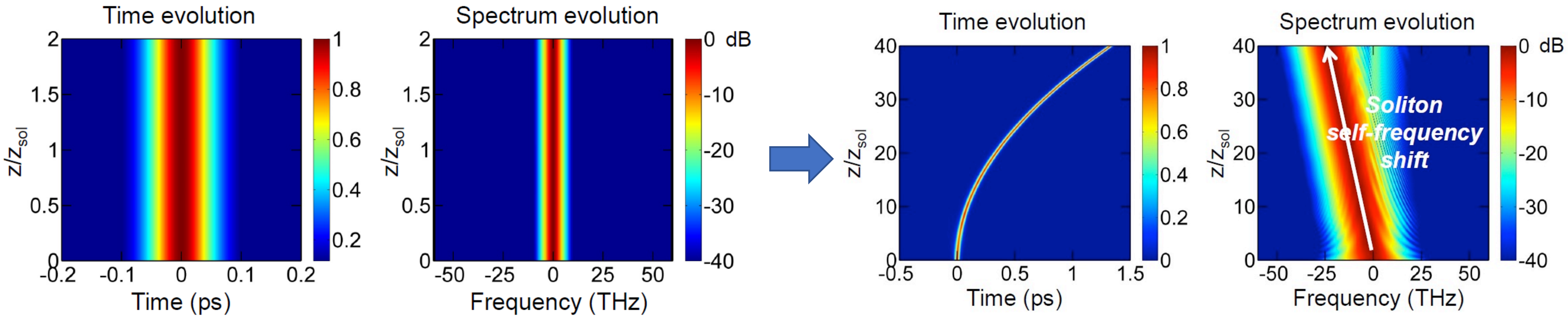
A quick glimpse into soliton dynamics – Raman pertubation

Raman perturbation:

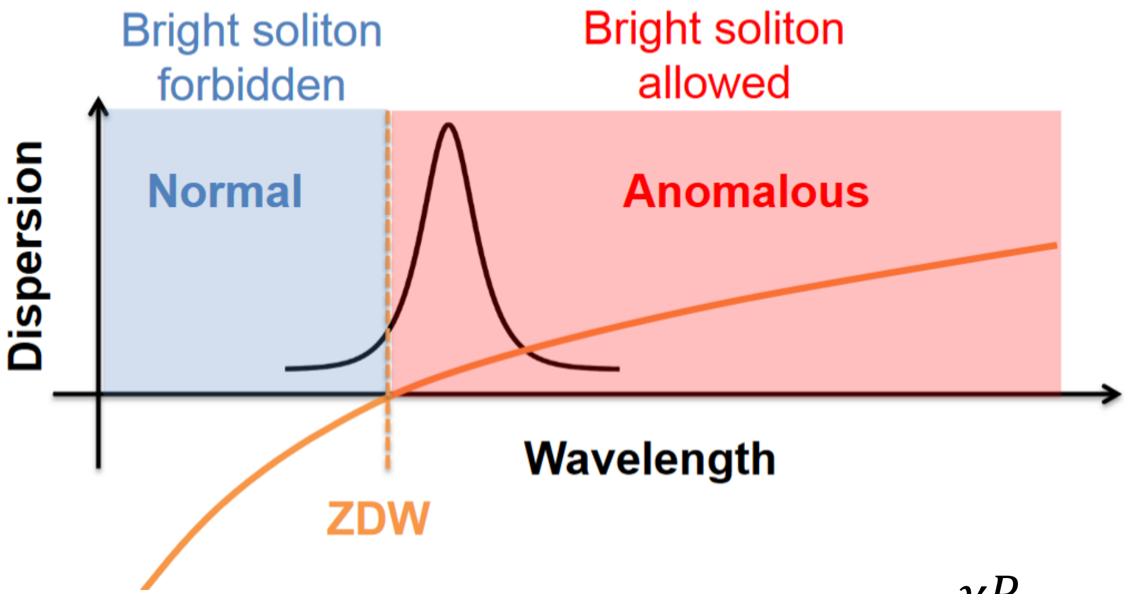


Mitschke and Mollenauer, OL 1986
Gordon OL 1986

- Parabolic trajectory in the time and frequency – soliton self frequency shift. The frequency shifts scales as $1/T_0^4$



A quick glimpse into soliton dynamics – HOD pertubation



If soliton is near the ZDW, it will be strongly perturbed:

- Part of its spectrum will extend in the normal dispersion regime

Soliton can shed part of its energy to a wave located in the normal regime if there is phase matching

- phase of the soliton = phase of the DW

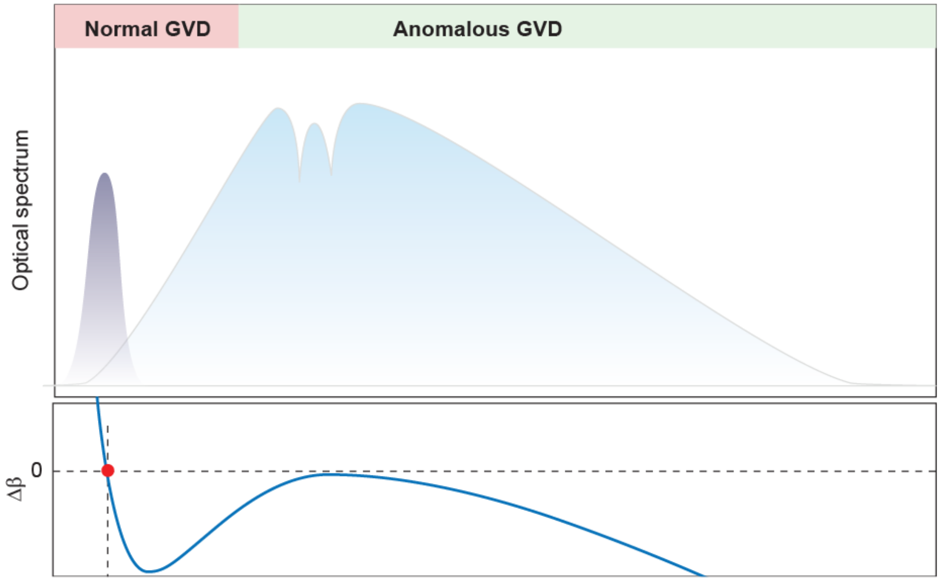
$$\beta(\omega_s) - v_g^{-1}\omega_s + \frac{\gamma P}{2} = \beta(\omega_d) - v_g^{-1}\omega_d$$

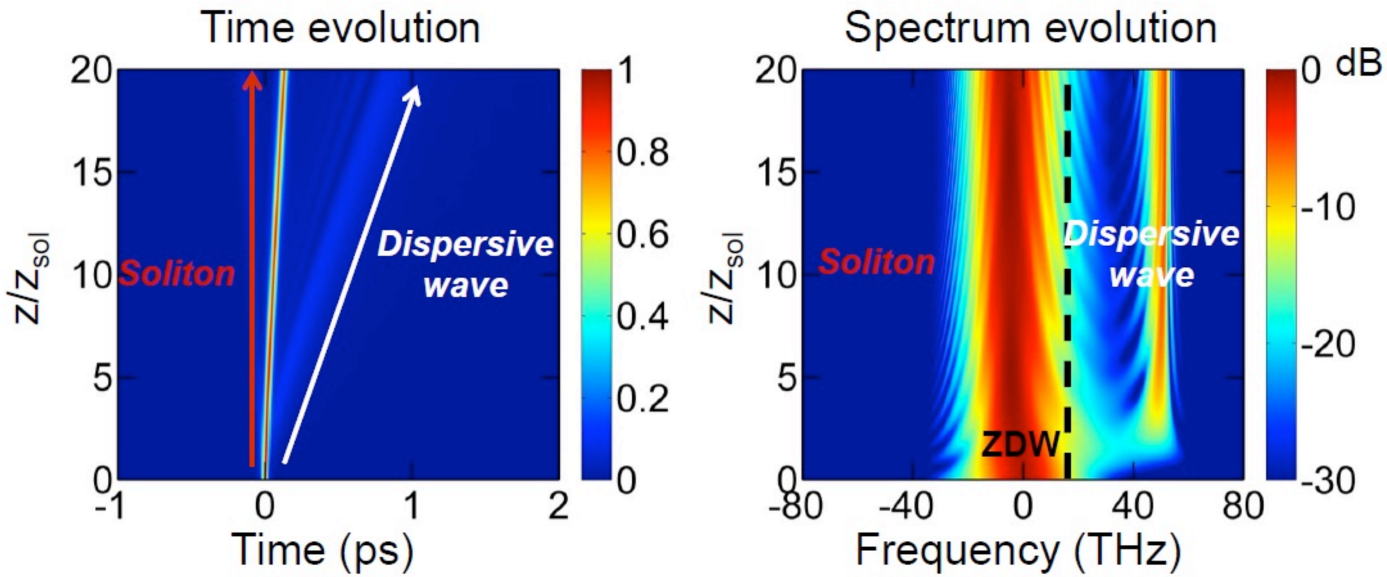
$$\Delta\beta(\omega) = \beta(\omega) - \beta(\omega_s) - v_g^{-1}(\omega - \omega_s) + \frac{\gamma P}{2}$$

$$\Delta\beta(\omega) \approx \sum_{m\geq 2} \frac{(\omega - \omega_s)^2}{m!} \frac{d^m}{d\omega^m}\beta(\omega)$$

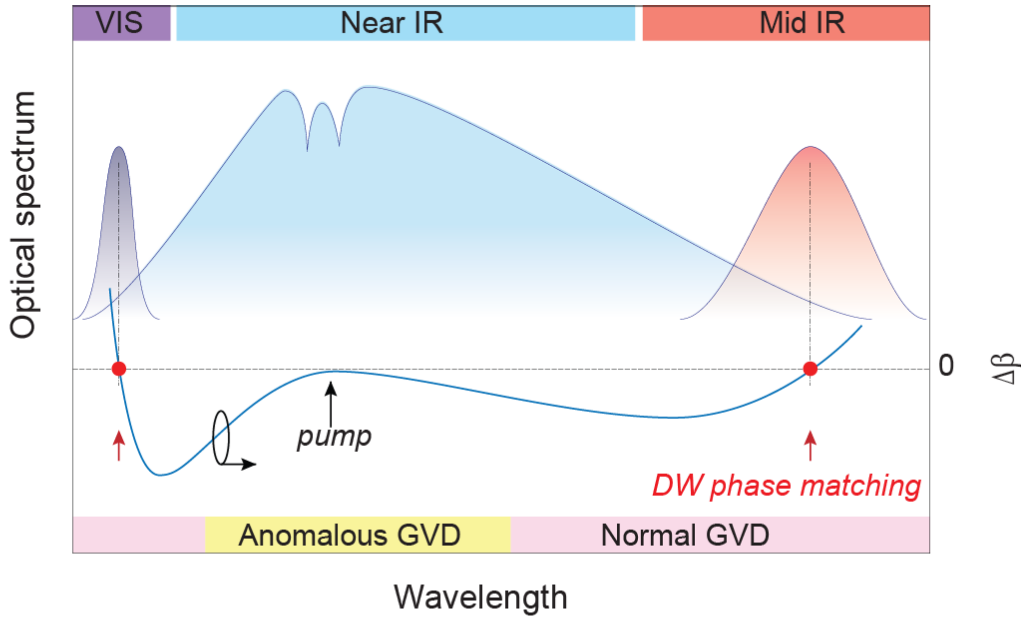
Way, Menyuk OL 1986
Akhmediev Phys Rev 1995

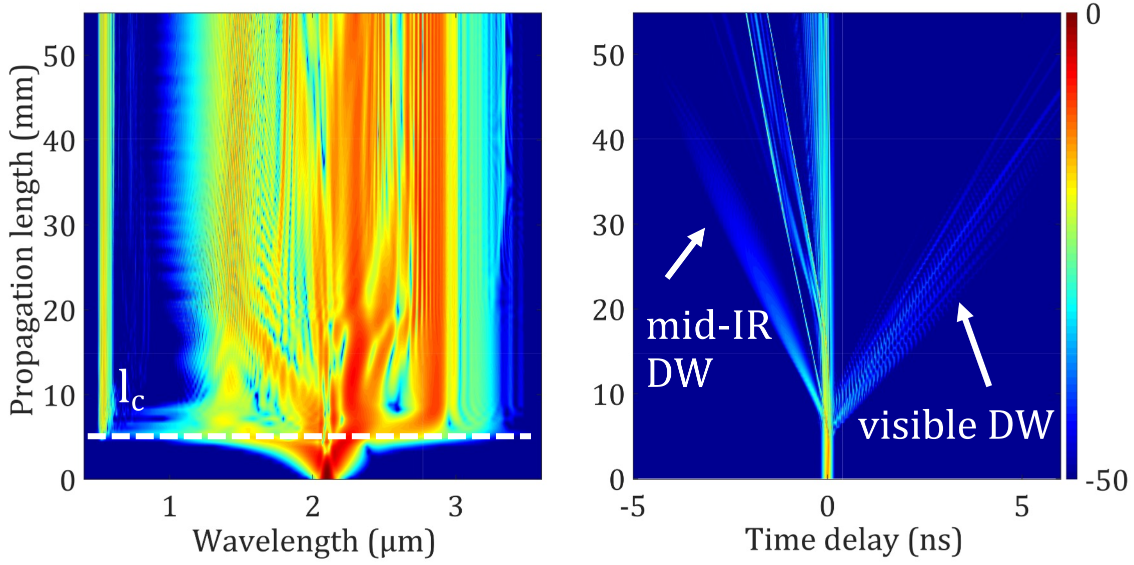
A quick glimpse into soliton dynamics – HOD pertubation





A quick glimpse into soliton dynamics – HOD pertubation

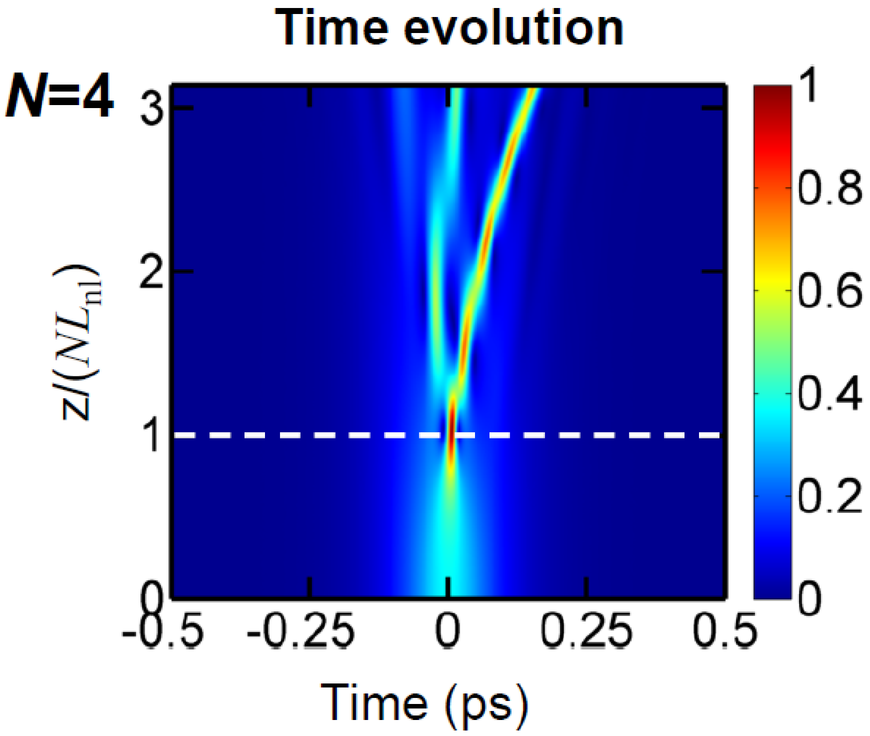




Higher order solitons

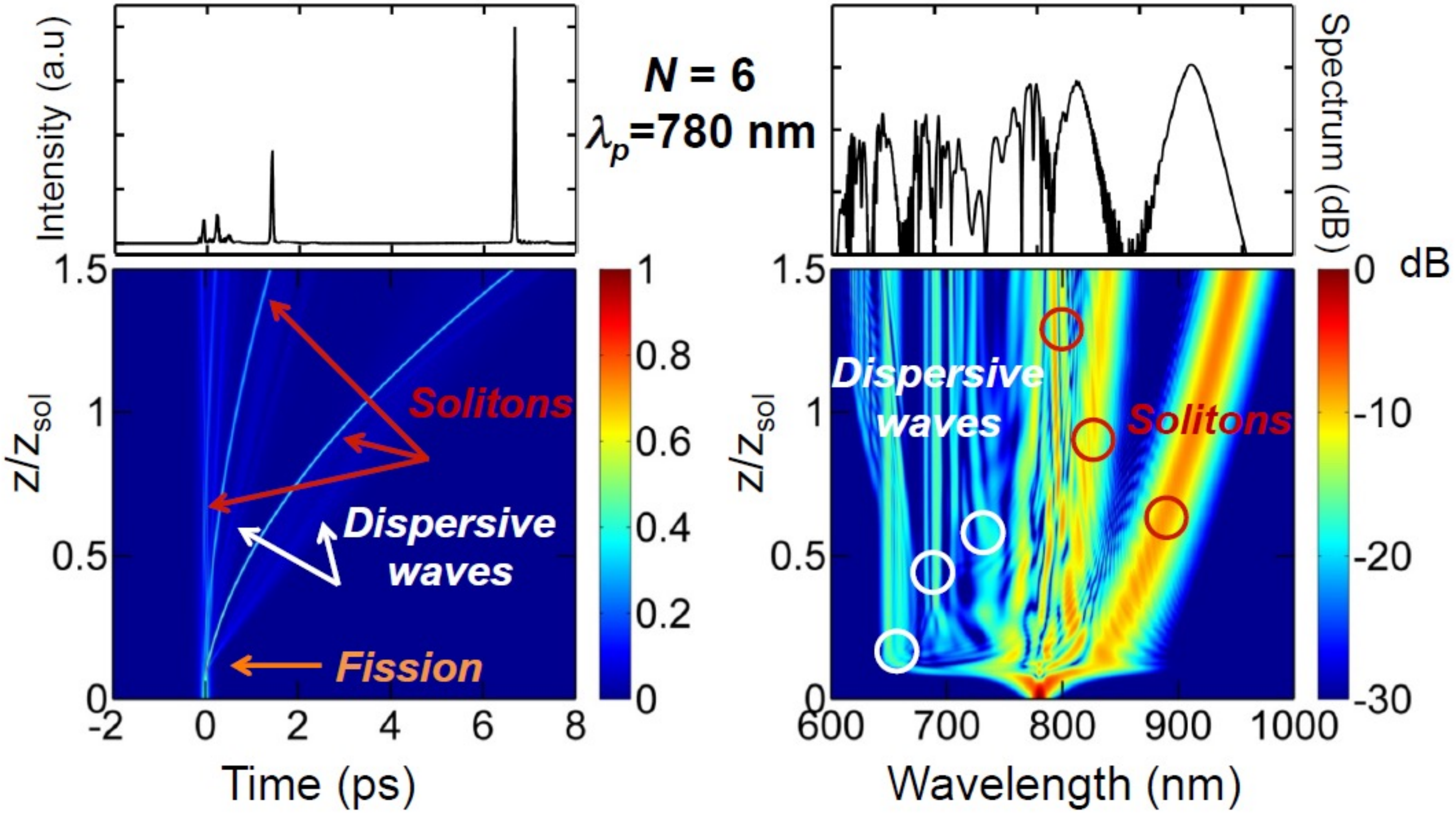
Higher order N-soliton is unstable and sensitive to these perturbations (Raman, HOD …)

- N soliton breaks unto N fundamental solitons: called soliton fission
- Multiple fundamental pulses are 'ejected'
- Fission length is approximately given by $L_{fiss} \approx \frac{L_D}{N} = \frac{T_0}{\sqrt{\gamma P_0 |\beta_2|}}$



All together

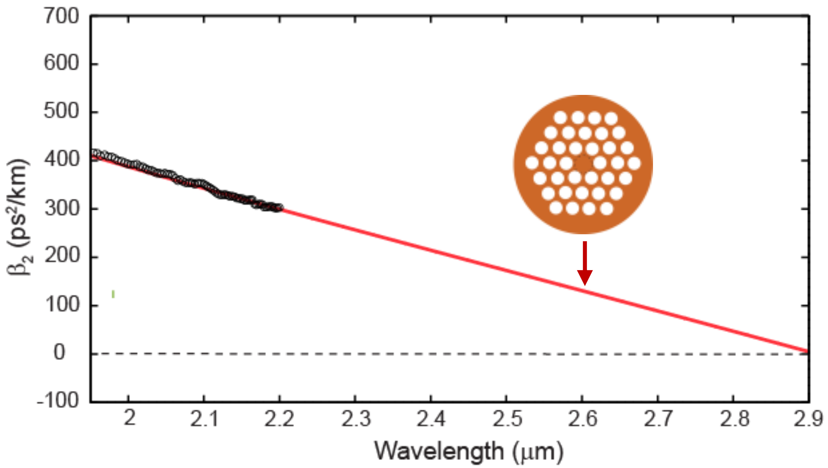
Soliton fission + DW generation + soliton self frequency shift = SCG

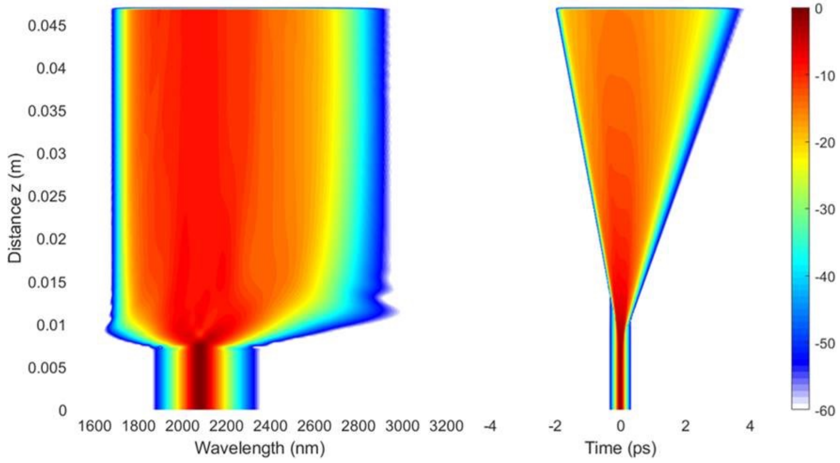


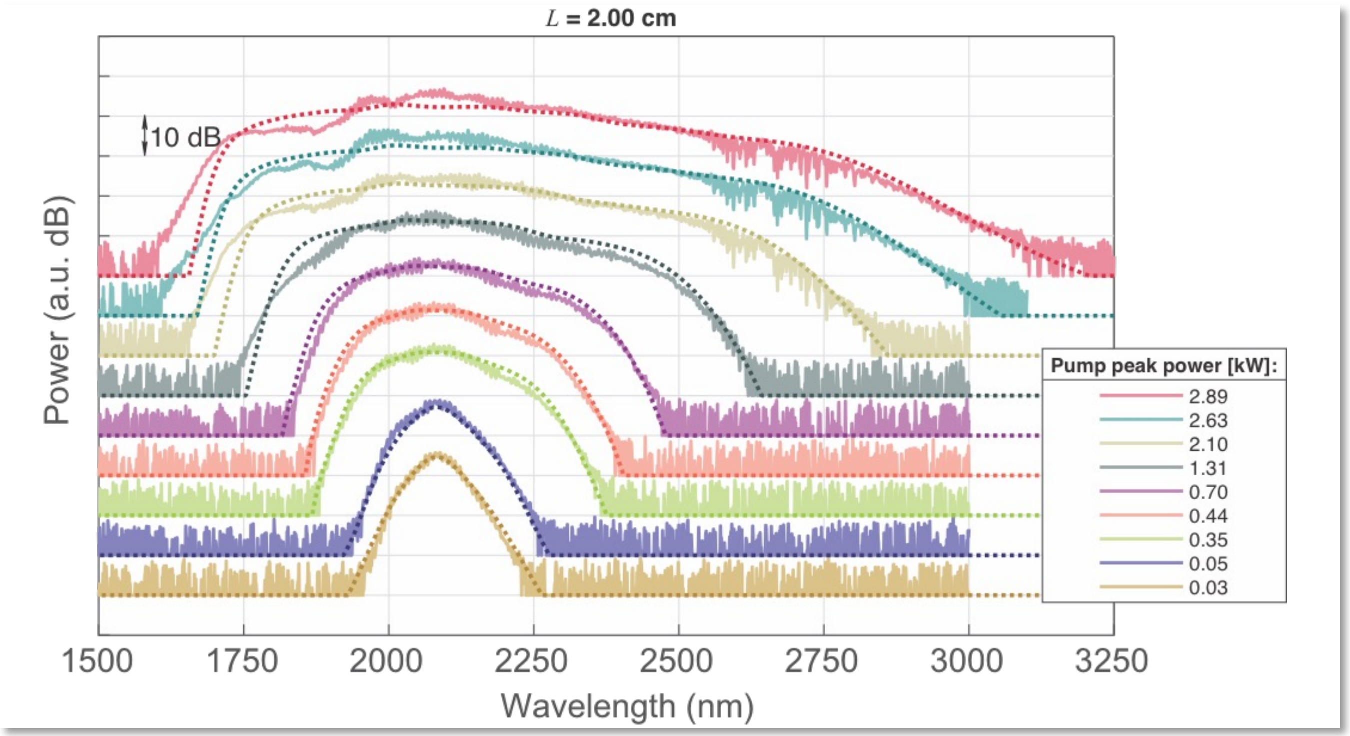
Short pulses and normal dispersion regime

There are no bright solitons in normal dispersion regime: the dynamics are completely different

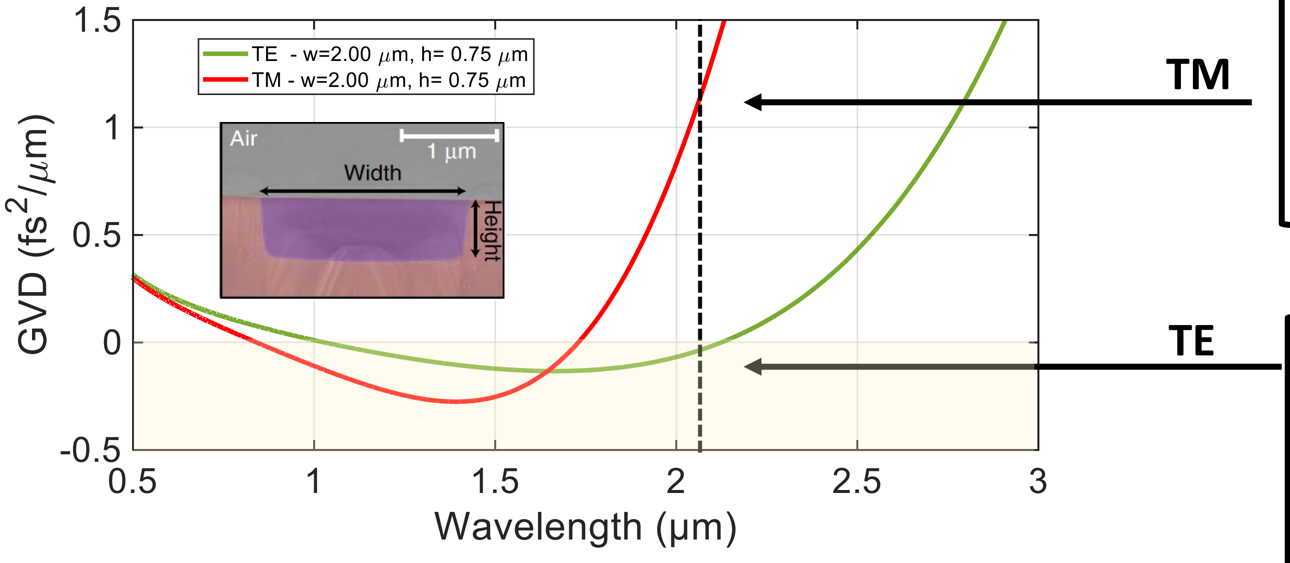
- It is dominated by SPM, optical wave breaking and FWM.



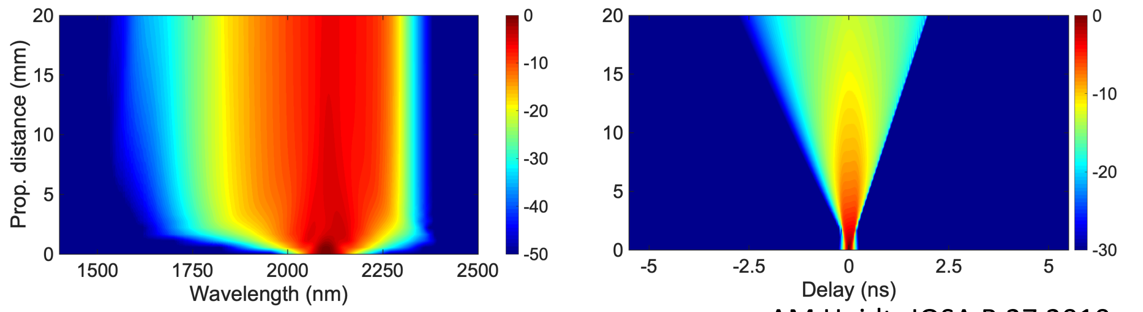




Further dispersion engineering: polarization selective SCG

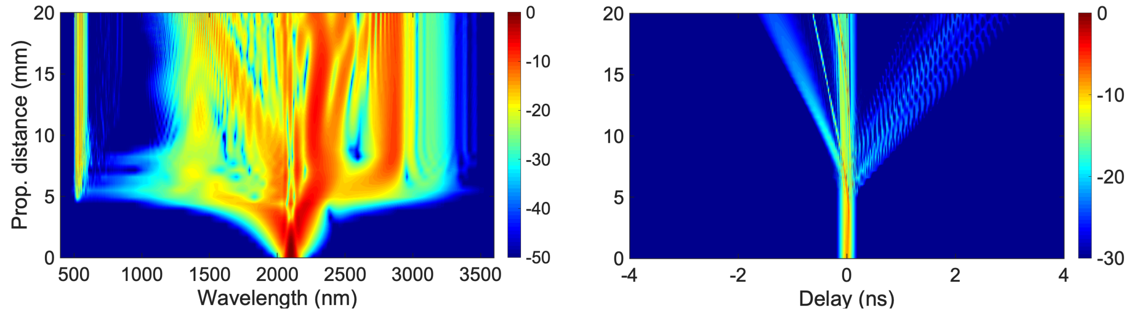


- All normal dispersion (ANDi) SCG



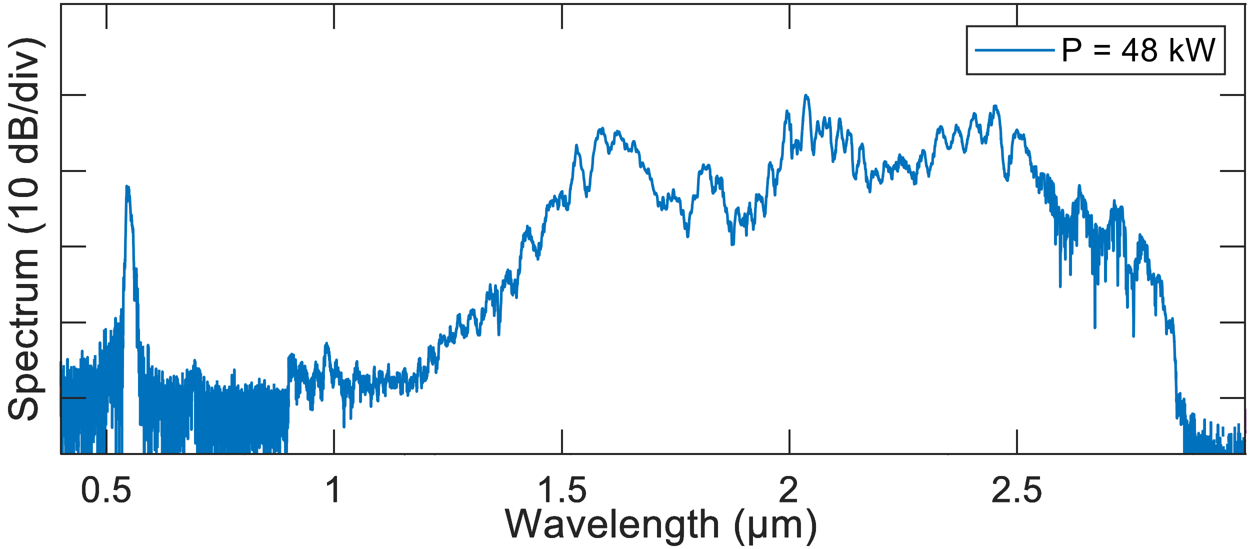
AM Heidt, JOSA B 27 2010

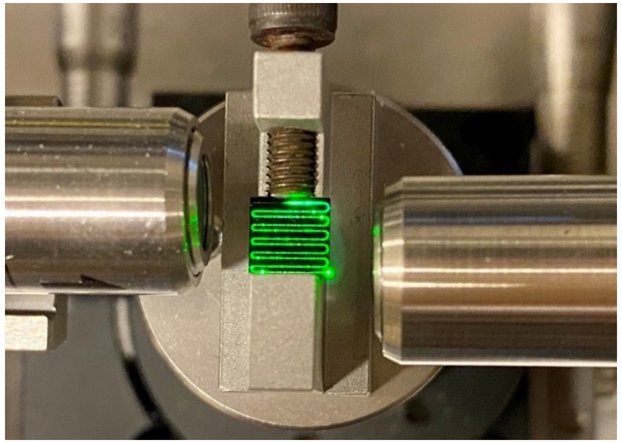
- Anomalous SCG



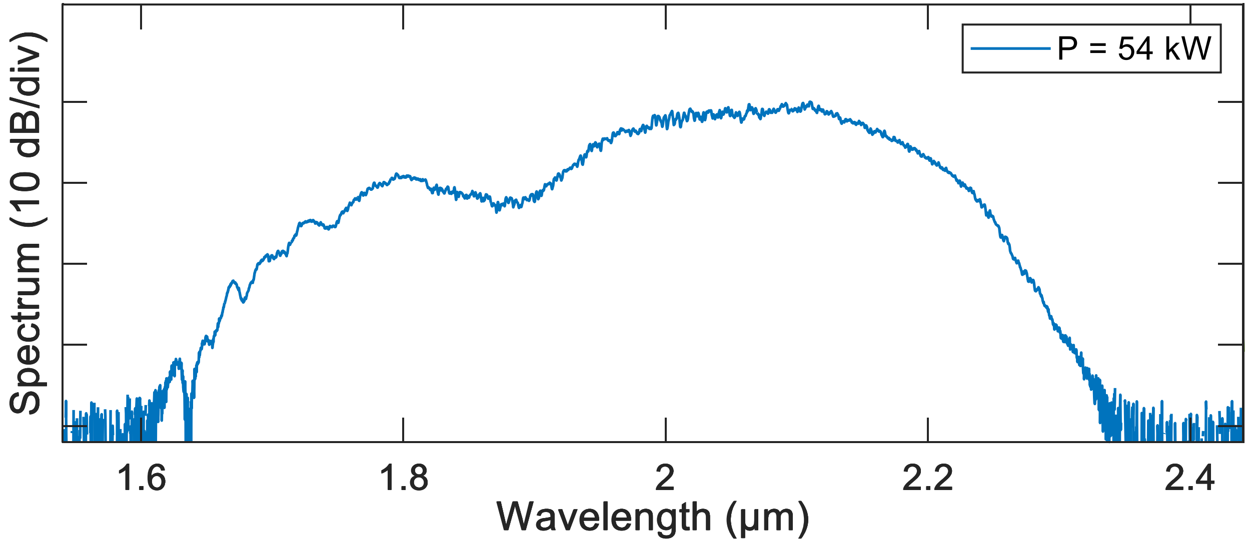
Polarization selective SCG

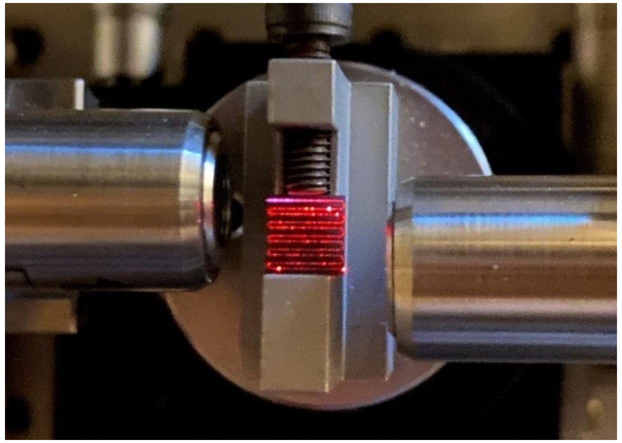
TE polarization





TM polarization

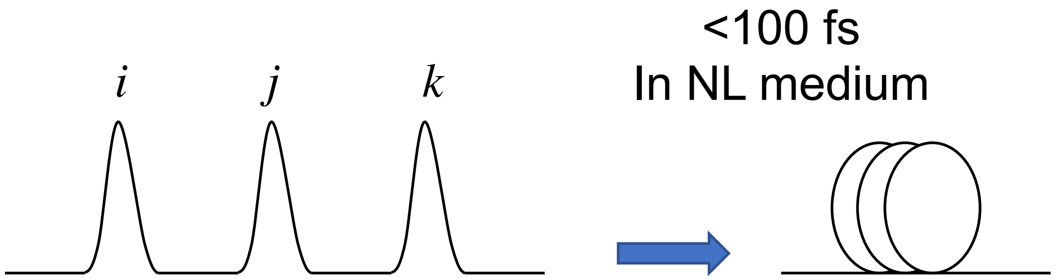




E. Tagkoudi et al., ***Optics Express*** 29(14) 2021

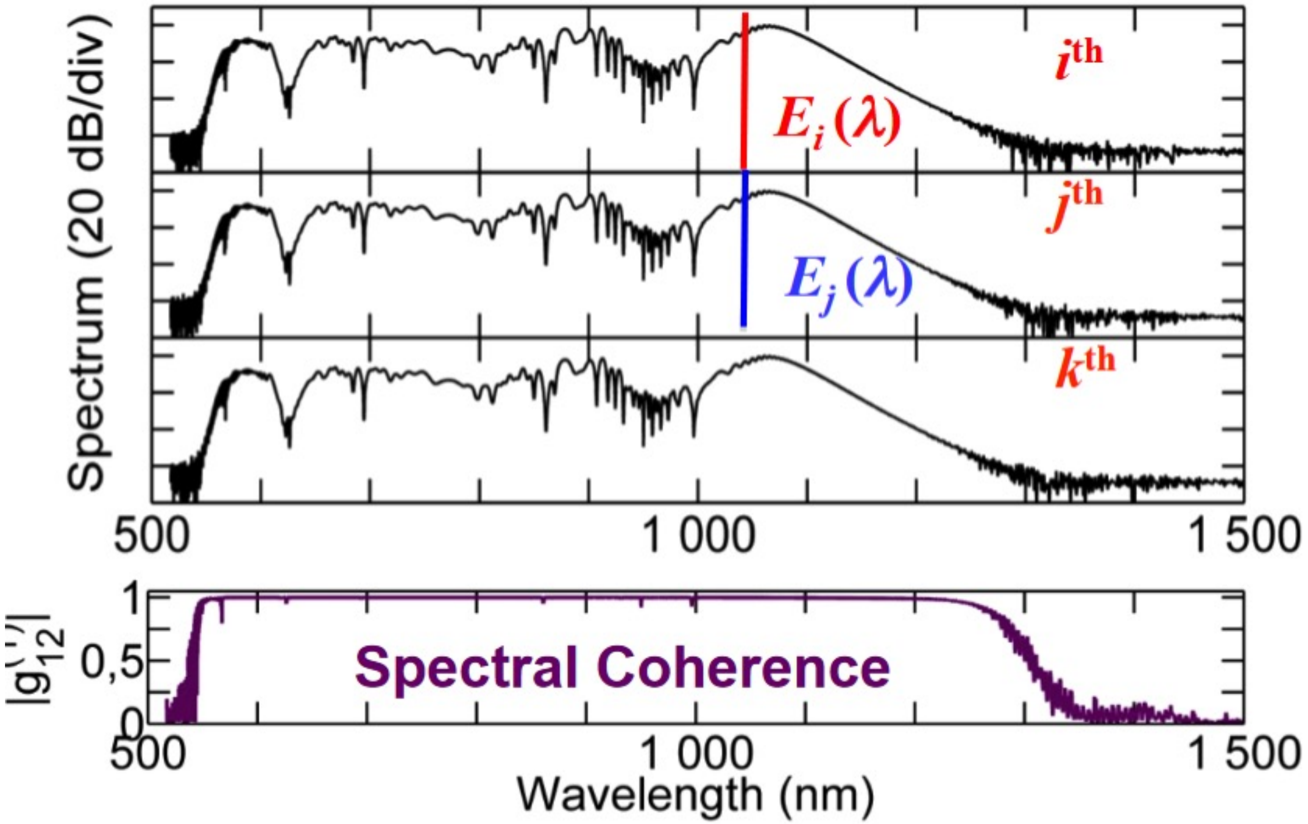
Coherence of supercontinuum

Spectral coherence: how spectra differs from shot to shot ?

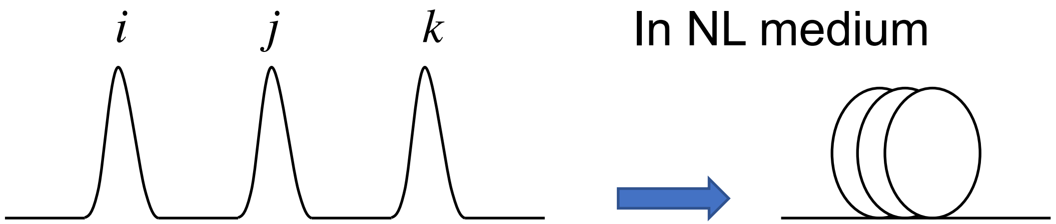


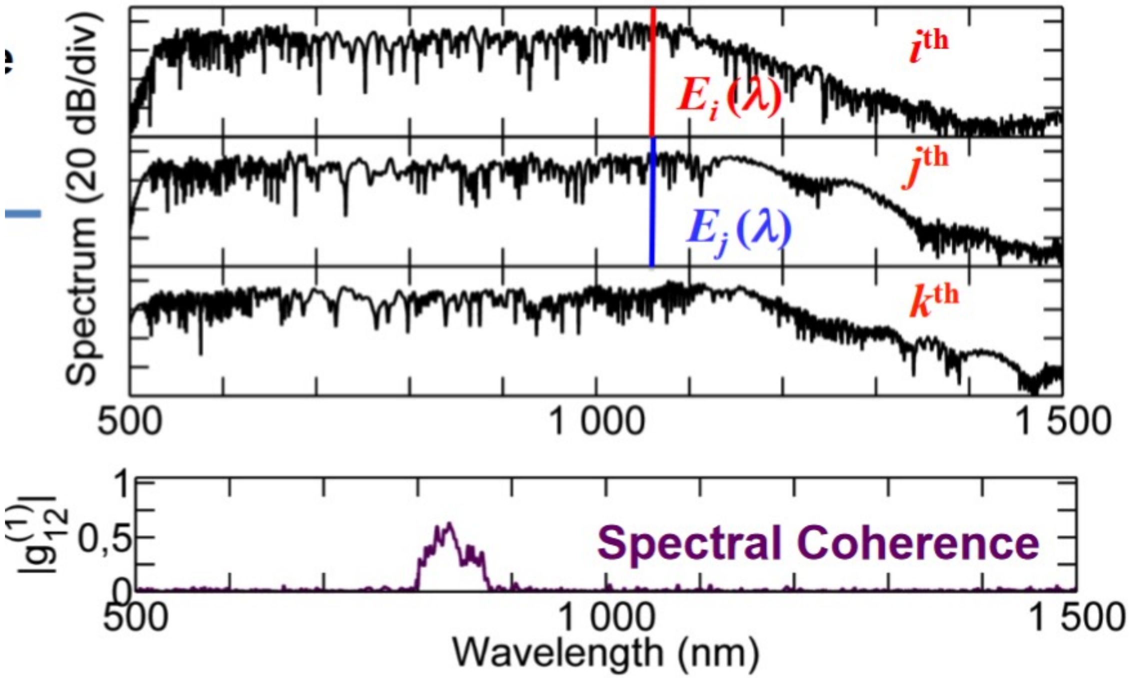
Complex degree of first-order coherence:

$$g_{12}^{(1)}(\lambda) = \frac{\left\langle E_i(\lambda) E_j^*(\lambda) \right\rangle_{i \neq j}}{\sqrt{\langle |E_i(\lambda)^2| \rangle \langle |E_j(\lambda)^2| \rangle}}$$



Coherence of supercontinuum

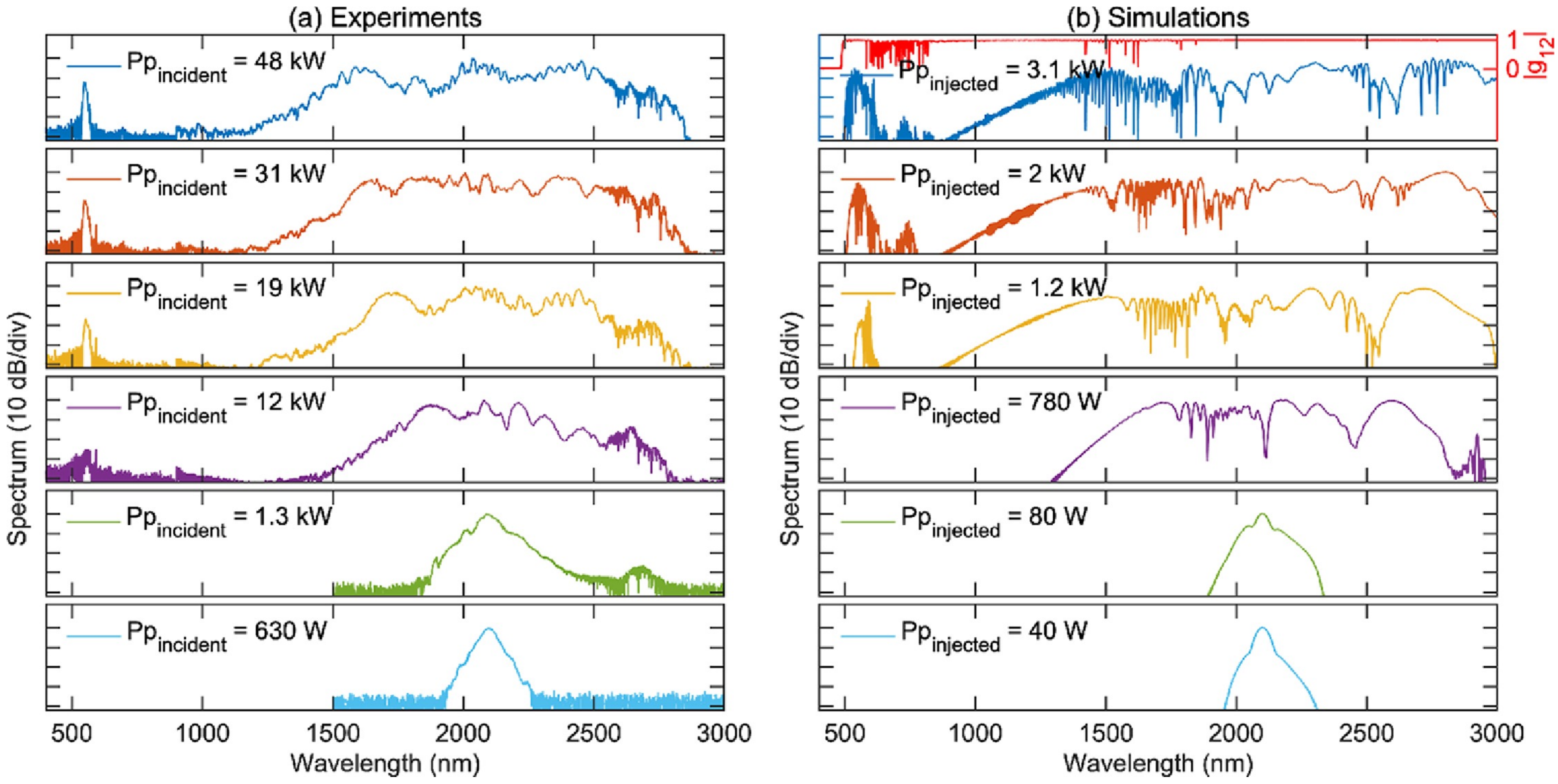




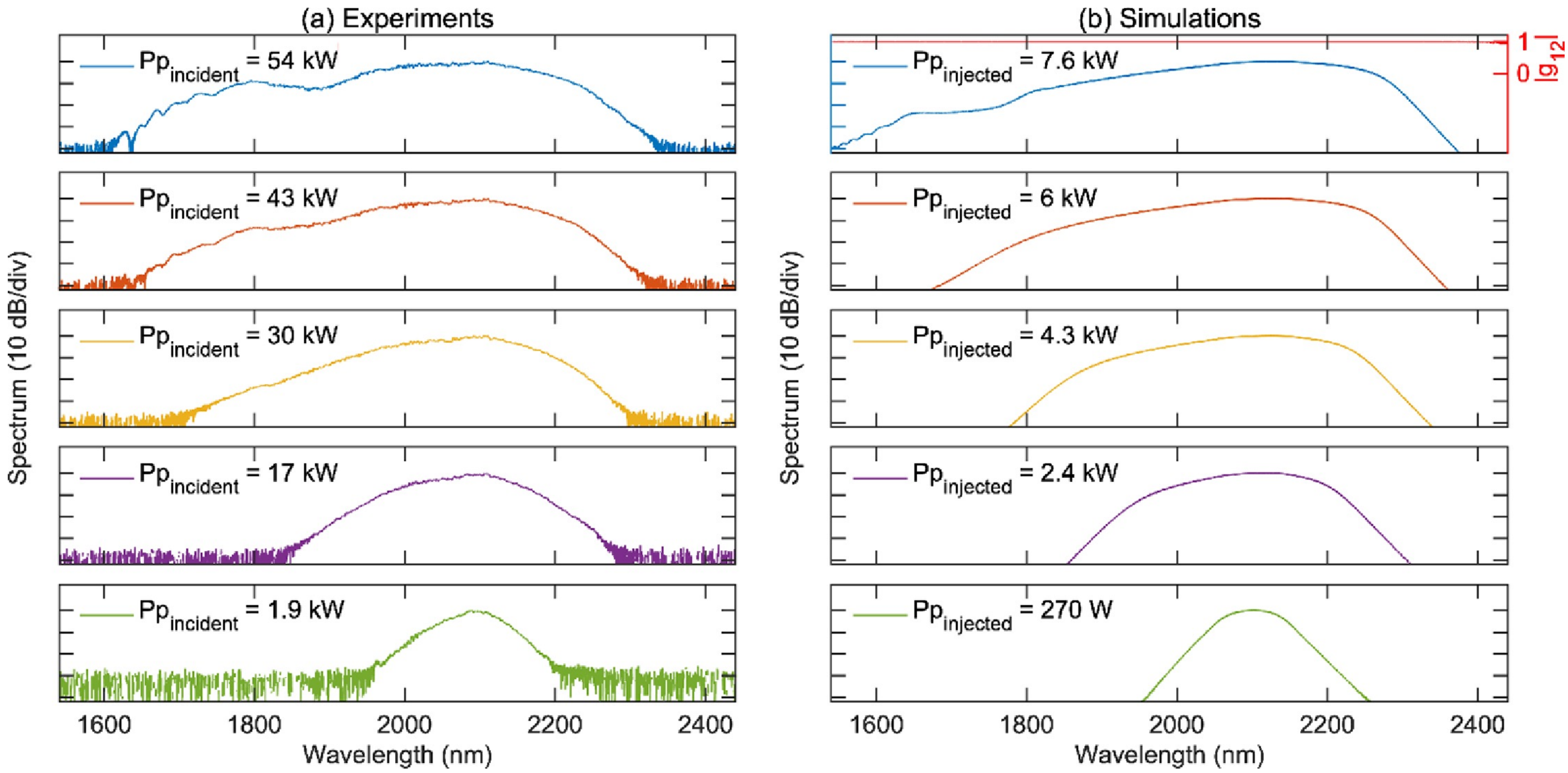
Supercontinuum is not necessarily coherent

- Can still find many use for imaging, sensing, spectroscopy etc ….

Example – anomalous dispersion



Example normal dispersion



Supercontinuum generation regimes

$$N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} \approx 16$$

	Long pulses (ns)	Short pulses (fs)
Anomalous dispersion	Mod[illegible]lation instability Sc[illegible]dynamics	Soli[illegible]ons Di[illegible]e waves
Normal dispersion	Raman scat[illegible]g Four-wave mixing	Self-phase m[illegible]dulation Four-wave mixing

INCOHERENT

COHERENT

Trends in SCG

New materials for fiber SCG

- ZBLAN
- Tellurite and chalcogenide PCFs

Towards on chip supercontinuum sources

- Chalcogenide waveguides
- Silicon nitride
- SiGe/Si
- Ge/Si ….

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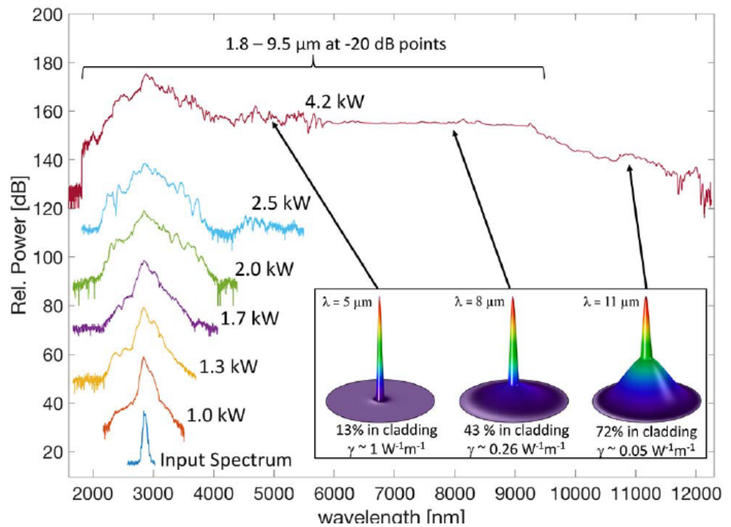
Toward all-fiber supercontinuum spanning the mid-infrared

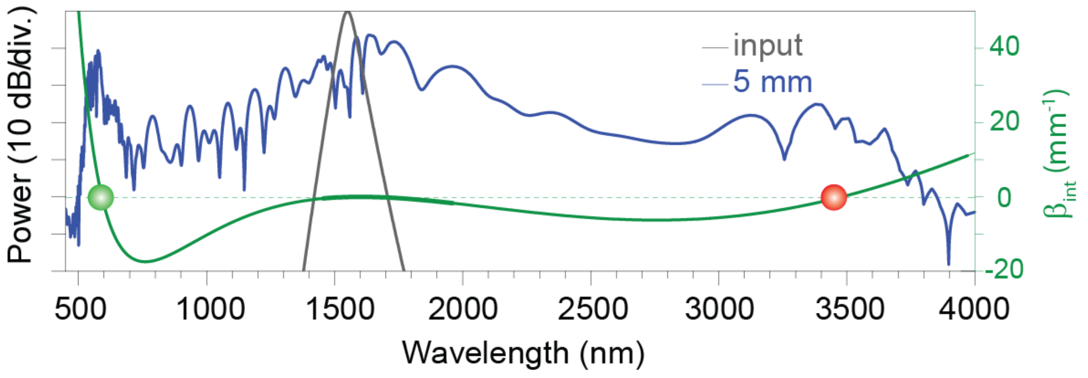
Darren D. Hudson,[1,*] Sergei Antipov,[1] Lizhu Li,[2] Imtiaz Alamgir,[2] Tomonori Hu,[3] Mohammed El Amraoui,[4] Younes Messaddeq,[4] Martin Rochette,[2] Stuart D. Jackson,[5] and Alexander Fuerbach[1]



ARTICLES
https://doi.org/10.1038/s41566-018-0144-1
nature photonics
Corrected: Publisher Correction

Mid-infrared frequency comb via coherent dispersive wave generation in silicon nitride nanophotonic waveguides

Hairun Guo[1,5], Clemens Herkommer[1,2,5], Adrien Billat[3,5], Davide Grassani[3], Chuankun Zhang[1,4], Martin H. P. Pfeiffer[1], Wenle Weng[1], Camille-Sophie Brès[3*] and Tobias J. Kippenberg[1*]



Summary of integrated SCG